TEMA 5. 1º BACHILLERATO A

1. a) Pasar a forma polar y trigonométrica: $-3-\sqrt{7}\,i$

   b) Pasar a forma binómica y trigonométrica: $\sqrt{5}_{185º}$

   c) Pasar a forma binómica y polar z= 6(cos 75º+i sen 75º) (1,5 puntos)

2. Calcular m y n para que se cumpla:

   $\frac{3m-5i}{-4+ni} = \sqrt{3}_{60º}$ (1,5 puntos)

3. Resuelve la ecuación $z^8 = -12+9i$ (1,5 puntos)

4. Calcula:

   $a)\ \frac{(-8-5i)(2+i)^5}{-3-7i} =$ b) $\frac{(5_{125º} \cdot 3_{65})^4}{(7_{36º})^5} =$ (1,5 puntos)

5. Resuelve (3-5i)(-6+2i)+8z= z(-2-9i)-4i

   (Hay que despejar z y tiene que ser un número complejo)(1 punto)

6. Calcula:

   $a)\ \frac{i^{17}-i^6+i^{25}-i^{21}}{5-i^{23}} =$ b) $\frac{5i^{13}-8i^5+(2i)^3}{-2i^{11}-i^{16}} =$ (1,5 puntos)

7. La suma de dos números complejos conjugados es 6 y la suma de sus módulos es 10. Determina cuales son los dos números. (1,5 puntos)

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① a) $z=-3-\sqrt{7}i$ (1,5)

$r=\sqrt{(-3)^2+(-\sqrt{7})^2}=\sqrt{9+7}=\sqrt{16}=4$

$\alpha=\text{arc tg}\dfrac{-\sqrt{7}}{-3}=221,41°$

Forma polar $z=4_{221,41°}$

Forma trigonométrica $z=4(\cos 221,41°+i\,\text{sen}\,221,41)$

b) $z=\sqrt{5}_{185°}$

$a=\sqrt{5}\cos 185°=-2,23$

$b=\sqrt{5}\,\text{sen}\,185°=-0,19$

Forma binómica: $z=-2,23-0,19i$

Forma trigonométrica $z=\sqrt{5}(\cos 185°+i\,\text{sen}\,185°)$

c) $z=6(\cos 75°+i\,\text{sen}\,75°)$

$a=6\cos 75°=1,55$

$b=6\,\text{sen}\,75°=5,8$

Forma polar $z=6_{75°}$

Forma binómica $z=1,55+5,8i=\dfrac{3\sqrt{6}-3\sqrt{2}}{2}+\dfrac{3\sqrt{6}+3\sqrt{2}}{2}i$

② (1,5)

$\sqrt{3}_{60°}=\sqrt{3}\cos 60°+i\sqrt{3}\,\text{sen}\,60°=\dfrac{\sqrt{3}}{2}+\dfrac{3}{2}i=0,87+1,5i$

$\dfrac{3m-5i}{-4+ni}=0,87+1,5i \Rightarrow (3m-5i)=(0,87+1,5i)(-4+ni)=$

$=-3,48+0,87ni-6i+1,5ni^2=$

$=(-3,48-1,5n)+(0,87n-6)i$

$\left.\begin{array}{l}3m=-3,48-1,5n\\ -5=0,87n-6\end{array}\right\}$ $n=\dfrac{-5+6}{0,87}=1,15$ $\boxed{n=1,15}=\dfrac{2\sqrt{3}}{3}$

$m=\dfrac{-3,48-1,5.(1,15)}{3}=-1,735$ $\boxed{m=-1,735}=-\sqrt{3}$

③ (1,5) $z^8=-12+9i=15_{143,13°}$

$r=\sqrt{(-12)^2+9^2}=\sqrt{144+81}=\sqrt{225}=15$

$\alpha=\text{arc tg}\dfrac{9}{-12}=143,13°$

$s=\sqrt[8]{15}$

$\beta\Rightarrow\begin{cases}\dfrac{360}{8}=45°\\ \dfrac{143,13°}{8}=17,89°\end{cases}$

$\sqrt[8]{15}_{17,89°}$, $\sqrt[8]{15}_{62,89°}$, $\sqrt[8]{15}_{107,89°}$, $\sqrt[8]{15}_{152,89°}$

$\sqrt[8]{15}_{197,89}$, $\sqrt[8]{15}_{242,89}$, $\sqrt[8]{15}_{287,89}$, $\sqrt[8]{15}_{332,89°}$

④ (1,5)

a) $\dfrac{(-8-5i)(2+i)^5}{-3-7i}=\dfrac{\sqrt{89}_{212}\cdot(\sqrt{5}_{26,57°})^5}{\sqrt{58}_{246,80°}}=\dfrac{\sqrt{89}_{212}\cdot\sqrt{3125}_{132,85°}}{\sqrt{58}_{246,80°}}=69,25_{98,05°}$

$z_1=-8-5i=\sqrt{89}_{212}$

$r=\sqrt{(-8)^2+(-5)^2}=\sqrt{64+25}=\sqrt{89}$

$\alpha=\text{arc tg}\dfrac{-5}{-8}=212°$

$z_2=2+i=\sqrt{5}_{26,57°}$

$r=\sqrt{4+1}=\sqrt{5}$

$\alpha=\text{arctg}\dfrac{1}{2}=26,57°$

$z_3=-3-7i=\sqrt{58}_{246,80°}$

$r=\sqrt{(-3)^2+(-7)^2}=\sqrt{58}$

$\alpha=\text{arc tg}\dfrac{-7}{-3}=246,80$

b) $\dfrac{(5_{125}\cdot 3_{65})^4}{(7_{360})^5} = \dfrac{(15_{190^\circ})^4}{(7_{36^\circ})^5} = \dfrac{50625_{760^\circ}}{16807_{180^\circ}} = 3{,}012_{580^\circ} = 3{,}012_{220^\circ}$

(5) (1)

$(3-5i)(-6+2i) + 8z = z(-2-9i) - 4i$

$-18 + 6i + 30i - 10i^2 + 8z = -2z - 9zi - 4i$

$-8 + 36i + 8z = -2z - 9zi - 4i$

$8z + 2z + 9zi = 8 - 36i - 4i$

$10z + 9zi = 8 - 40i$

$z(10+9i) = 8 - 40i$

$z = \dfrac{8-40i}{10+9i} = \dfrac{(8-40i)(10-9i)}{(10+9i)(10-9i)} = \dfrac{80 - 72i - 400i + 360i^2}{100 - 81i^2} =$

$= \dfrac{(80-360) + (-72-400)i}{100+81} = \dfrac{-280-472i}{181} = -\dfrac{280}{181} - \dfrac{472}{181}i$

(6) (1,5)

a) $\dfrac{i^{17} - i^6 + i^{25} - i^{21}}{5 - i^{23}} = \dfrac{i - i^2 + i - i}{5 - i^3} = \dfrac{i+1}{5+i} = \dfrac{(1+i)(5-i)}{(5+i)(5-i)} = \dfrac{5 - i + 5i - i^2}{25 - i^2} =$

$= \dfrac{6+4i}{26} = \dfrac{6}{26} + \dfrac{4}{26}i = \dfrac{3}{13} + \dfrac{2}{13}i$

b) $\dfrac{5i - 8i + 8i^3}{-2i^3 - i^0} = \dfrac{-3i - 8i}{+2i - 1} = \dfrac{-11i(-1-2i)}{(-1+2i)(-1-2i)} = \dfrac{-11i(-1-2i)}{1 - 4i^2} =$

$= \dfrac{11i + 22i^2}{1+4} = \dfrac{-22 + 11i}{5} = -\dfrac{22}{5} + \dfrac{11}{5}i$

(7) (1,5)

$z_1 = a + bi$

$z_2 = a - bi$

$z_1 + z_2 = 6$

$|z_1| + |z_2| = 10$

$(a+bi) + (a-bi) = 6 \rightarrow 2a = 6 \Rightarrow a = 3$

$\sqrt{a^2+b^2} + \sqrt{a^2+(-b)^2} = 10$

$\sqrt{9+b^2} + \sqrt{9+b^2} = 10 \Rightarrow 2\sqrt{9+b^2} = 10 \Rightarrow$

$\left(\sqrt{9+b^2}\right)^2 = 5^2 \Rightarrow 9 + b^2 = 25 \rightarrow b^2 = 25 - 9 = 16$

$b = \pm 4$

Los números son $z_1 = 3 + 4i$

$z_2 = 3 - 4i$