(3) Dadas las matrices $A=\begin{pmatrix} -2 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ y $B=\begin{pmatrix} 4 & 1 & -1 \\ 0 & 2 & 1 \\ 2 & 0 & -3 \end{pmatrix}$.

Resuelve el sistema matricial $\left.\begin{array}{l} 3X-4Y=A \\ 5X+2Y=B \end{array}\right\}$

$$\left.\begin{array}{l} 3X-4Y=A \\ 5X+2Y=B \end{array}\right\} \quad \begin{array}{rl} \cdot(5) & \cancel{15X}-20Y=5A \\ \cdot(-3) & \underline{\cancel{-15X}-6Y=-3B} \\ & -26Y=5A-3B \end{array} \rightarrow Y=\frac{5A-3B}{-26}$$

$$\begin{array}{rl} \cdot & 3X-\cancel{4Y}=A \\ \cdot(2) & \underline{10X+\cancel{4Y}=2B} \\ & 13X=A+2B \end{array} \longrightarrow X=\frac{A+2B}{13}$$

$$X=\frac{1}{13}\left[\begin{pmatrix} -2 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix}+\begin{pmatrix} 8 & 2 & -2 \\ 0 & 4 & 2 \\ 4 & 0 & -6 \end{pmatrix}\right]=\begin{pmatrix} 6/13 & 3/13 & -3/13 \\ 0 & 3/13 & 3/13 \\ 4/13 & 1/13 & -3/13 \end{pmatrix}$$

$$Y=-\frac{1}{26}\left[\begin{pmatrix} -10 & 5 & -5 \\ 0 & -5 & 5 \\ 0 & 5 & 15 \end{pmatrix}-\begin{pmatrix} 12 & 3 & -3 \\ 0 & 6 & 3 \\ 6 & 0 & -9 \end{pmatrix}\right]=\begin{pmatrix} 11/13 & -1/13 & 1/13 \\ 0 & 11/26 & -1/13 \\ 3/13 & -5/26 & -12/13 \end{pmatrix}$$

(4) Calcula todas las matrices B que conmutan con la matriz $A=\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$$A\cdot B=B\cdot A \rightarrow \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} a+2c & b+2d \\ c & d \end{pmatrix}=\begin{pmatrix} a & 2a+b \\ c & 2c+d \end{pmatrix}$$

$$\begin{array}{l} a+2c=a \rightarrow \boxed{c=0} \\ b+2d=2a+b \rightarrow 2a=2d \rightarrow \boxed{a=d} \\ c=c \\ d=2c+d \rightarrow \boxed{c=0} \end{array} \qquad B=\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \quad \forall a,b\in\mathbb{R}$$

(5) Dadas las matrices $A=\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & -1 \\ 5 & -1 & 2 \end{pmatrix}$, $B\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ resuelve $AXA^t=B$

$$AXA^t=B \longrightarrow X=A^{-1}\cdot B\,(A^t)^{-1}=A^{-1}\cdot B\cdot(A^{-1})^t$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -3 & 1 & -1 & 0 & 1 & 0 \\ 5 & -1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\substack{3F_1+F_2 \\ 5F_1-F_3}]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 5 & 0 & -1 \end{array}\right) \xrightarrow[F_2-F_3]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 3 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right)$$

$$\xrightarrow[F_2+F_3]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \Longrightarrow A^{-1}=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & 1 \end{pmatrix}$$

$$X=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ -2 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}=$$

$$=\begin{pmatrix} 1 & 1 & -2 \\ 1 & -3 & -4 \\ -2 & -4 & 3 \end{pmatrix}$$