## CONTROL LÍMITES 1º BACHILLERATO B

1. Estudia la continuidad de la siguiente función:

$$f(x)=\begin{cases} \frac{5}{x-5} & si\ x\le 0 \\ \sqrt{x+1} & si\quad 0<x\le 3 \\ \frac{x-1}{2x-6} & si\ x>3 \end{cases}$$

2. Averiguar a,b para que la función f(x) sea continua:

$$f(x)=\begin{cases} 2x^2+10+b & si\ x<2 \\ x^2-3x+a & si\ \ 2\le x\le 6 \\ 5x-b & si\ x>6 \end{cases}$$

3. Calcula todas las asíntotas de las siguientes funciones:
   c. $f(x)=\frac{3+x-x^2}{x^2-1}$
   d. $f(x)=\frac{x^2+x-2}{x-3}$

4. Calcula los siguientes límites:
   a. $\lim_{x\to 0}\frac{3x^2-5x}{4x}$
   b. $\lim_{X\to\infty}\frac{X}{1-\sqrt{X+1}}$
   c. $\lim\limits_{x\to\infty}\left(\frac{7x^2-5x}{7x^2+2x-3}\right)^{\frac{x^2-5}{3x-8}}$
   d. $\lim\limits_{x\to -2}\frac{x^2+7x+10}{6x^2+3x}$
   e. $\lim\limits_{x\to 4}\frac{2x-8}{\sqrt{x+5}-3}$
   f. $\lim\limits_{x\to\infty}\left(\frac{3x^2-5x}{2x+3}-\frac{x^2+7}{3x-5}\right)$

① $\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}\cdot \ln x - \int -\frac{1}{x}\cdot\frac{1}{x}\,dx = -\frac{\ln x}{x} + \int x^{-2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + k$

$$\boxed{\begin{array}{l} u = \ln x \to du = \frac{1}{x}\,dx \\ dv = \int x^{-2}\,dx \to v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array}}$$

② $\int .\ \mathrm{arc\,tg}\,(2x+3)\,dx = x\ .\ \mathrm{arc\,tg}\,(2x+3) - \int x\ .\ \frac{2}{1+(2x+3)^2}\,dx = x\ \mathrm{arc\,tg}\,(2x+3) -$

$$\boxed{\begin{array}{l} u = \mathrm{arc\,tg}\,(2x+3) \to du = \frac{2}{1+(2x+3)^2}\,dx \\ dv = dx \Rightarrow v = x \end{array}}$$

$\int x\ .\ \frac{2}{1+(2x+3)^2}\,dx =$

$(2x+3)^2+1 = 4x^2+12x+9+1 = 4x^2+12x+10$

$= x\ \mathrm{arc\,tg}\,(2x+3) - \int \frac{2x}{4x^2+12x+10} =$

$= x\ \mathrm{arc\,tg}\,(2x-3) - \frac{1}{4}\int \frac{8x+12-12}{4x^2+12x+10}\,dx = x\ \mathrm{arc\,tg}\,(2x+3) - \frac{1}{4}\left[\int \frac{8x+12}{4x^2+12x+10}\,dx + \right.$

$\left. + \int \frac{-12}{4x^2+12x+10}\,dx\right] = x\ \mathrm{arc\,tg}\,(2x+3) - \frac{1}{4}\ln|4x^2+12x+10| + 3\ \mathrm{arc\,tg}\,(2x+3) +$

③ $\int \frac{2x+1}{x^2-3x+2}\,dx = \int \frac{-3}{x-1}\,dx + \int \frac{5}{x-2}\,dx = -3\ln|x-1| + 5\ln|x-2| + k$

$\frac{2x+1}{x^2-3x+2} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)} \Rightarrow \begin{array}{ll} x=2 & 5=B \\ x=1 & 3=-A \to A=-3 \end{array}$

④ $\int \mathrm{sen}^3 x\,dx = -\int \mathrm{sen}^2 x.(-\mathrm{sen}\,x)\,dx = -\int (1-t^2).\,dt = -t + \frac{t^3}{3} = -\cos x + \frac{\cos^3 x}{3} + k$

$\cos x = t$

$-\mathrm{sen}\,x\,dx = dt$

⑤ $\int \frac{x^3-3x^2+2x-5}{x-2}\,dx = \int \left(x^2 - x + \frac{-5}{x-2}\right)dx = \frac{x^3}{3} - \frac{x^2}{2} - 5\cdot\ln|x-2| + k$

$$\begin{array}{l|l} \cancel{x^3} - 3x^2 + 2x - 5 & x-2 \\ \hline -\cancel{x^3} + 2x^2 & x^2 - x \\ \quad -x^2 + 2x & \\ \quad +x^2 - 2x & \\ \qquad\quad -5 & \end{array}$$

⑥ $\int \frac{e^x}{1-e^{-x}}\,dx = \int \frac{1}{1-\frac{1}{t}}\,dt = \int \frac{t}{t-1}\,dt = \int\left(1 + \frac{1}{t-1}\right)dt = t + \ln|t-1| =$

$= e^x + \ln|e^x - 1| + k$

$e^x = t$

$e^x dx = dt$

$$\begin{array}{l|l} t & t-1 \\ \hline -t+1 & 1 \\ \quad 1 & \end{array}$$

⑧ $f(x) = \frac{x}{1+x^2} = 0 \to x = 0$

$\int_1^2 \frac{x}{1+x^2}\,dx = \frac{1}{2}\ln|1+x^2|\Big|_1^2 = \frac{1}{2}[\ln 5 - \ln 2]\ u^2$

⑦ $\int \frac{2x}{\sqrt{4-9x^4}}\,dx = \int \frac{x}{\sqrt{1-\frac{9x^4}{4}}}\,dx = \int \frac{x}{\sqrt{1-\left(\frac{3}{2}x^2\right)^2}}\,dx = \frac{1}{3}\int \frac{3x}{\sqrt{1-\left(\frac{3}{2}x^2\right)^2}}\,dx =$

$= \frac{1}{3}\ \mathrm{arc\,sen}\left(\frac{3}{2}x^2\right) + k.$

(9) $x^3 - x + 2 = -x^2 + x + 2$

$x^3 + x^2 - 2x = 0 \rightarrow x(x^2+x-2) = 0$

$x(x-1)(x+2) = 0 \rightarrow x = 0$

$\hookrightarrow x = 1$

$\hookrightarrow x = -2$

$$\left|\int_{-2}^{0}(x^3+x^2-2x)\,dx\right| + \left|\int_0^1 (x^3+x^2-2x)\,dx\right| = \left|\left.\frac{x^4}{4}+\frac{x^3}{3}-x^2\right|_{-2}^{0}\right| +$$

$$+\left|\left.\frac{x^4}{4}+\frac{x^3}{3}-x^2\right|_0^1\right| = \left|0-(4-\frac{8}{3}-4)\right| + \left|(\frac{1}{4}+\frac{1}{3}-1)-0\right| = \frac{8}{3}+\frac{5}{12} = \frac{37}{12}\,u^2$$

(10) $$\left|\int_{-2}^{-1}(-3x+2)\,dx\right| + \left|\int_{-1}^{2} 2^x\,dx\right| + \left|\int_2^3 \frac{3}{x}\,dx\right| =$$

$$\left|\left.-\frac{3x^2}{2}+2x\right|_{-2}^{1}\right| + \left|\left.\frac{2^x}{\ln 2}\right|_{-1}^{2}\right| + \left|\left.3\ln|x|\right|_2^5\right| = \frac{21}{2}+\frac{7/2}{\ln 2}+3\,|\ln 5-\ln 2|$$

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(1) Utilizar el cambio de variable $t^6 = 1+x$ para calcular $\int \frac{\sqrt{x+1}+2}{(x+1)^{2/3}-\sqrt{x+1}}\,dx$

$t^6 = 1+x \Rightarrow t = \sqrt[6]{1-x}$

$6t^5\,dt = dx$

$$\int \frac{\sqrt{x+1}+2}{(x+1)^{2/3}-\sqrt{x+1}}\,dx = \int \frac{\sqrt{t^6}+2}{(t^6)^{2/3}-\sqrt{t^6}}\cdot 6t^5\,dt = \int \frac{t^3+2}{t^4-t^3}\cdot 6t^5\,dt = 6\int \frac{t^5(t^3+2)}{t^4-t^3}\,dt =$$

$$= 6\int \frac{t^2(t^3+2)}{t-1}\,dt = 6\int \frac{t^5+2t^2}{t-1}\,dt \underset{\substack{\uparrow \\ \text{División}}}{=} 6\int\left(t^4+t^3+t^2+3t+3+\frac{3}{t-1}\right)dt =$$

$$= 6\left[\frac{t^5}{5}+\frac{t^4}{4}+\frac{t^3}{3}-\frac{3t^2}{2}+3t+3\ln|t-1|\right] =$$

$$= 6\left[\frac{\sqrt[6]{(x+1)^5}}{5}+\frac{\sqrt[6]{(x+1)^4}}{4}+\frac{\sqrt[6]{(x+1)^3}}{3}-\frac{3\sqrt[6]{(x+1)^2}}{2}+3\sqrt[6]{(x+1)}+3\ln\left|\sqrt[6]{x+1}-1\right| + k\right.$$

(2) $$\int_0^{\pi/2}\frac{\operatorname{sen} 2x}{1+\cos^2 2x}\,dx = -\frac{1}{2}\int_0^{\pi/2}\frac{-2\operatorname{sen} 2x}{1+(\cos 2x)^2}\,dx = -\frac{1}{2}\left.\operatorname{arc\,tg}(\cos 2x)\right|_0^{\pi/2} =$$

$$= -\frac{1}{2}\left[\operatorname{arc\,tg}(-1) - \operatorname{arc\,tg}(1)\right] = -1\left(-\frac{\pi}{2}-\frac{\pi}{2}\right) = -(-\pi) = \pi$$

También se puede hacer con un cambio de variable

$\cos 2x = t \Rightarrow -2\operatorname{sen} 2x\,dx = dt \Rightarrow \operatorname{sen} 2x\,dx = -\frac{dt}{2}$

$$\int_0^{\pi/2}\frac{-\frac{1}{2}}{1+t^2}\,dt = -\frac{1}{2}\left.\operatorname{arc\,tg}\, t\right|_0^{\pi/2} = -\frac{1}{2}\left.\operatorname{arc\,tg}(\cos 2\pi)\right|_0^{\pi/2}\cdots$$