CONTROL TEMA 1. 2º BACHILLERATO A 2020 (2)

1. Sean las matrices $A=\begin{pmatrix}1 & 0\\ 2 & k\\ 0 & 1\end{pmatrix}$ $y\, B=\begin{pmatrix}k & 0 & -1\\ 1 & 1 & 2\end{pmatrix}$
   a. Estudia en función de los valores reales de k, si la matriz B·A tiene inversa. Calcúlala si es posible para k=1.
   b. Estudia en función de los valores reales de k, si la matriz A·B tiene inversa.

2. Calcula A^n, A^3, A^6 $siendo\, A=\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 0\\ 0 & 0 & 2\end{pmatrix}$

3. Dadas las matrices $A=\begin{pmatrix}-1 & 0 & 2\\ 3 & 1 & -1\\ 4 & 3 & -6\end{pmatrix}$ $y\, B=\begin{pmatrix}1 & -1 & 0\\ 2 & 2 & 5\\ -3 & 4 & 0\end{pmatrix}$.

   Resuelve el sistema matricial $\begin{cases}2X-5Y=A\\ 4X+3Y=B\end{cases}$

4. Calcula todas las matrices B que conmutan con la matriz $A=\begin{pmatrix}1 & -1\\ 4 & 3\end{pmatrix}$

5. Dadas las matrices $A=\begin{pmatrix}2 & 1 & 0\\ -1 & 0 & 0\\ 1 & 2 & -1\end{pmatrix}$, $B=\begin{pmatrix}-1 & 0 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{pmatrix}$ $y\, C=\begin{pmatrix}0 & 1 & 0\\ 0 & 3 & 0\\ -1 & 0 & -1\end{pmatrix}$, resuelve la siguiente ecuación 2X+C = A – X·B

TEMA 1. 2º BACH A

① $A = \begin{pmatrix} 1 & 0 \\ 2 & k \\ 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} k & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix}$

a) $\underset{(2\times3)\,(3\times2)}{B \cdot A} = (2\times2)$ $B \cdot A = \begin{pmatrix} k & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} k & -1 \\ 3 & k+2 \end{pmatrix}$

$$\begin{pmatrix} k & -1 \\ 3 & k+2 \end{pmatrix} \xrightarrow[3F_1 - kF_2]{} \begin{pmatrix} k & -1 \\ 0 & -3-k^2-2k \end{pmatrix}$$

$-k^2-2k-3 \neq 0 \quad \forall k \in \mathbb{R} \quad rg\ B\cdot A = 2 = \text{orden}$

Tiene inversa para cualquier valor de k

Si $k=1$

$$\left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 3 & 3 & 0 & 1 \end{array}\right) \xrightarrow[3F_1-F_2]{} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & -6 & 3 & -1 \end{array}\right) \xrightarrow[6F_1-F_2]{} \left(\begin{array}{cc|cc} 6 & 0 & 3 & 1 \\ 0 & -6 & 3 & -1 \end{array}\right)$$

$$\xrightarrow[\substack{F_1/6 \\ F_2/-6}]{} \left(\begin{array}{cc|cc} 1 & 0 & 1/2 & 1/6 \\ 0 & 1 & -1/2 & 1/6 \end{array}\right) \qquad (B\cdot A)^{-1} = \begin{pmatrix} 1/2 & 1/6 \\ -1/2 & 1/6 \end{pmatrix}$$

Comprobación $\begin{pmatrix} 1 & -1 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 1/2 & 1/6 \\ -1/2 & 1/6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

b) $\underset{(3\times2)\,(2\times3)}{A \cdot B} = (3\times3)$

$$A \cdot B = \begin{pmatrix} 1 & 0 \\ 2 & k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} k & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} k & 0 & -1 \\ 3k & k & -2+2k \\ 1 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} k & 0 & -1 \\ 3k & k & -2+2k \\ 1 & 1 & 2 \end{pmatrix} \xrightarrow[\substack{3F_1-F_2 \\ F_1-kF_3}]{} \begin{pmatrix} k & 0 & -1 \\ 0 & -k & -1-2k \\ 0 & -k & -1-2k \end{pmatrix} \xrightarrow[F_2-F_3]{} \begin{pmatrix} k & 0 & -1 \\ 0 & -k & -1-2k \\ 0 & 0 & 0 \end{pmatrix}$$

$rg\ A\cdot B = 2 \quad \forall k \in \mathbb{R}$, como $rg\ A\cdot B \neq$ orden $A\cdot B$ no tiene inversa para ningún valor.

② $A^2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

$A^3 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 7 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

$A^4 = \begin{pmatrix} 1 & 3 & 7 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 15 \\ 0 & 1 & 0 \\ 0 & 0 & 16 \end{pmatrix}$

$A^n = \begin{pmatrix} 1 & n & 2^n-1 \\ 0 & 1 & 0 \\ 0 & 0 & 2^n \end{pmatrix}$

$A^6 = \begin{pmatrix} 1 & 6 & 63 \\ 0 & 1 & 0 \\ 0 & 0 & 64 \end{pmatrix}$

③ $A = \begin{pmatrix} -1 & 0 & 2 \\ 3 & 1 & -1 \\ 4 & 3 & -6 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 2 & 5 \\ -3 & 4 & 0 \end{pmatrix} \Rightarrow \begin{cases} 2X - 5Y = A \\ 4X + 3Y = B \end{cases}$

$\cdot(-2)\quad \cancel{-4X} + 10Y = -2A$

$\cdot\quad \cancel{4X} + 3Y = B$

$Y = -\frac{2A+B}{13}$

$\cdot(3)\quad 6X - \cancel{15Y} = 3A$

$\cdot(5)\quad 20X + \cancel{15Y} = 5B$

$X = \frac{3A+5B}{26}$

$$X = \frac{\begin{pmatrix} -3 & 0 & 6 \\ 9 & 3 & -3 \\ 12 & 9 & -18 \end{pmatrix} + \begin{pmatrix} 5 & -5 & 0 \\ 10 & 10 & 25 \\ -15 & 20 & 0 \end{pmatrix}}{26} = \begin{pmatrix} 2/26 & -5/26 & 6/26 \\ 19/26 & 13/26 & 22/26 \\ -3/26 & 29/26 & -18/26 \end{pmatrix}$$

$$Y = \frac{\begin{pmatrix} 2 & 0 & -4 \\ -6 & -2 & 2 \\ -8 & -6 & +12 \end{pmatrix} + \begin{pmatrix} 1 & -1 & 0 \\ 2 & 2 & 5 \\ -3 & 4 & 0 \end{pmatrix}}{13} = \begin{pmatrix} 3/13 & -1/13 & -4/13 \\ -4/13 & 0 & 7/13 \\ -11/13 & -2/13 & 12/13 \end{pmatrix}$$

(4) $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $AB = B \cdot A$

$$\begin{pmatrix} 1 & -1 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 4 & 3 \end{pmatrix}$$

$$\begin{pmatrix} a-c & b-d \\ 4a+3c & 4b+3d \end{pmatrix} = \begin{pmatrix} a+4b & -a+3b \\ c+4d & -c+3d \end{pmatrix}$$

$$\left.\begin{array}{l} a-c = a+4b \\ b-d = -a+3b \\ 4a+3c = c+4d \\ 4b+3d = -c+3d \end{array}\right\} \begin{array}{l} -c = 4b \\ a = 2b+d \\ 4a = 4d-2c \rightarrow 4a = 4d+8b \rightarrow a = 2b+d \\ -c = 4b \qquad\qquad d = a-2b \end{array}$$

$$\begin{pmatrix} a & b \\ -4b & a-2b \end{pmatrix} \quad \forall a,b \in \mathbb{R}$$

También puede ser

$$\begin{pmatrix} a & \frac{a-d}{2} \\ -2a+2d & d \end{pmatrix} \quad \forall a,d \in \mathbb{R}$$

(5) $2X + C = A - X \cdot B \longrightarrow 2X + XB = A - C$

$$X(2I+B) = A-C$$

$$X = (A-C)(2I+B)^{-1}$$

$$2I+B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\substack{2F_1 - F_2 \\ F_1 - F_3}]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 2 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 \end{array}\right)$$

$$\xrightarrow[\substack{F_1 + F_3 \\ F_2 + 2F_3}]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & -1 & 0 & 4 & -1 & -2 \\ 0 & 0 & -1 & 1 & 0 & -1 \end{array}\right) \xrightarrow[\substack{F_2/-1 \\ F_3/-1}]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & -4 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$(2I+B)^{-1} = \begin{pmatrix} 2 & 0 & -1 \\ -4 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

$$A-C = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 1 & 2 & -1 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 0 & 3 & 0 \\ -1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ -1 & -3 & 0 \\ 2 & 2 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & 0 & 0 \\ -1 & -3 & 0 \\ 2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & -1 \\ -4 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & -2 \\ 10 & -3 & -5 \\ -4 & 2 & 2 \end{pmatrix}$$