TEMA 2 2º BACHILLERATO A

1. Escribe las propiedades de los determinantes
2. Calcula el siguiente determinante:

$$\begin{vmatrix} -1 & 1 & -2 & 1 & -6 \\ 3 & -1 & 4 & -3 & 0 \\ 1 & -3 & 0 & -2 & 2 \\ 6 & -2 & -2 & -1 & 1 \\ 2 & -2 & 1 & -1 & 0 \end{vmatrix}$$

3. Sea la matriz

$$A = \begin{pmatrix} 1 & m & -1 & 3 \\ m & 1 & 2 & m \\ -6 & 3 & -14 & m \end{pmatrix}$$

Estudia el rango de A en función de los valores de m.

4. Se consideran las matrices A=$\begin{pmatrix} k & 2 & 1 \\ k & 3 & 2 \\ 1 & k & -4 \end{pmatrix}$; B=$\begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -2 \\ 2 & 4 & 1 \end{pmatrix}$

   a. Obtener los valores de k para los que A es singular.
   b. Halle, si es posible, la matriz inversa de A en el caso de k=0 y resuelva AX-X=B

5. Sean A y B dos matrices cuadradas de orden 3, cuyos determinantes son $|A| = 4 \quad y \quad |B| = -2$. Si A=$(C_1, C_2, C_3)$. Calcular:

a) $|A^t B^4|$ b) $|A^{-1} \cdot 3B|$

c) $|D| = |5C_2 + 2C_3 - 3C_1, -2C_2 + C_1, -5C_3 + 4C_1|$

TEMA 2

(2)
$$\begin{vmatrix} -1 & 1 & -2 & 1 & -6 \\ 3 & -1 & 4 & -3 & 0 \\ 1 & -3 & 0 & -2 & 2 \\ 6 & -2 & -2 & -1 & 1 \\ 2 & -2 & 1 & -1 & 0 \end{vmatrix} \underset{\substack{F_1=F_1+6F_4 \\ F_3=F_3-2F_4}}{=} \begin{vmatrix} 35 & -11 & -14 & -5 & 0 \\ 3 & -1 & 4 & -3 & 0 \\ -11 & 1 & +4 & 0 & 0 \\ \cancel{6} & \cancel{-2} & \cancel{-2} & \cancel{-1} & 1 \\ 2 & -2 & 1 & -1 & 0 \end{vmatrix} =$$

$$- \begin{vmatrix} 35 & -11 & -14 & -5 \\ 3 & -1 & 4 & -3 \\ -11 & 1 & +4 & 0 \\ 2 & -2 & 1 & -1 \end{vmatrix} \underset{\substack{C_1=C_1-2C_3 \\ C_2=C_2+2C_3 \\ C_4=C_4+C_3}}{=} - \begin{vmatrix} 63 & -39 & -14 & -19 \\ -5 & 7 & 4 & 1 \\ -19 & 9 & 4 & 4 \\ \cancel{0} & \cancel{0} & 1 & \cancel{0} \end{vmatrix} = \begin{vmatrix} 63 & -39 & -19 \\ -5 & 7 & 1 \\ -19 & 9 & 4 \end{vmatrix} =$$

$$= 3360 - (3874) = -514$$

(3)
$$A = \begin{pmatrix} 1 & m & -1 & 3 \\ m & 1 & 2 & m \\ -6 & 3 & -14 & m \end{pmatrix}$$

$$\begin{vmatrix} 1 & m & -1 \\ m & 1 & 2 \\ -6 & 3 & -14 \end{vmatrix} = -14 - 3m - 12m - 6 - 6 + 14m^2 = 14m^2 - 15m - 26 = 0$$

$$m = \frac{15 \pm 41}{28} \begin{cases} m_1 = 2 \\ m_2 = -\frac{26}{28} = -\frac{13}{14} \end{cases}$$

$m = (-15/8)$

Si $m \neq 2, -13/14 \Rightarrow rg\ A = 3$

Si $m = 2$
$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 1 & 2 & 2 \\ -6 & 3 & -14 & 2 \end{pmatrix} \qquad \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & 2 \\ 3 & -14 & 2 \end{vmatrix} = 0$$

$rg\ A = 2$

Si $m = -13/14$
$$\begin{pmatrix} 1 & -13/14 & -1 & 3 \\ -13/14 & 1 & 2 & -13/14 \\ -6 & 3 & -14 & -13/14 \end{pmatrix} \qquad \begin{vmatrix} -13/14 & -1 & 3 \\ 1 & 2 & -13/14 \\ 3 & -14 & -13/14 \end{vmatrix} = -15138$$

$rg\ A = 3$

Si $m \neq 2$ $rg\ A = 3$, si $m = 2$ $rg\ A = 2$.

(4) $A = \begin{pmatrix} k & 2 & 1 \\ k & 3 & 2 \\ 1 & k & -4 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -2 \\ 2 & 4 & 1 \end{pmatrix}$

a) $\begin{vmatrix} k & 2 & 1 \\ k & 3 & 2 \\ 1 & k & -4 \end{vmatrix} = -12k + k^2 + 4 - 3 - 2k^2 + 8k = -k^2 - 4k + 1 = 0$

$k = -2 \pm \sqrt{5}$

Si $k = \dfrac{4 \pm \sqrt{20}}{-2}$ → la matriz es singular.

b) $k=0$ $A = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & -4 \end{pmatrix}$ $|A| = 1$

$A_{11} = -12$ $A_{21} = +8$ $A_{31} = 1$

$A_{12} = +2$ $A_{22} = -1$ $A_{32} = 0$

$A_{13} = -3$ $A_{23} = +2$ $A_{33} = 0$

$A^{-1} = \begin{pmatrix} -12 & 8 & 1 \\ 2 & -1 & 0 \\ -3 & 2 & 0 \end{pmatrix}$

$AX - X = B \Rightarrow (A - I)X = B \Rightarrow X = (A-I)^{-1}B$

$C = A - I = \begin{pmatrix} -1 & 2 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & -5 \end{pmatrix}$ $|C| = 14 - 2 = 12$

$C_{11} = -10$ $C_{21} = 10$ $C_{31} = 2$

$C_{12} = 2$ $C_{22} = 4$ $C_{32} = +2$

$C_{13} = -2$ $C_{23} = +2$ $C_{33} = -2$

$C^{-1} = \frac{1}{12}\begin{pmatrix} -10 & 10 & 2 \\ 2 & 4 & 2 \\ -2 & 2 & -2 \end{pmatrix}$

$X = \frac{1}{12}\begin{pmatrix} -10 & 10 & 2 \\ 2 & 4 & 2 \\ -2 & 2 & -2 \end{pmatrix}\begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -2 \\ 2 & 4 & 1 \end{pmatrix} = \frac{1}{12}\begin{pmatrix} -6 & 38 & -48 \\ 6 & 14 & 0 \\ -6 & -2 & -12 \end{pmatrix} = \begin{pmatrix} -1/2 & 19/6 & -4 \\ 1/2 & 7/6 & 0 \\ -1/2 & -1/6 & 1 \end{pmatrix}$

(5) a) $|A^t B^4| = |A^t| \cdot |B^4| = |A| \cdot |B|^4 = 4 \cdot (-2)^4 = 4 \cdot 16 = 64$

b) $|A^{-1} \cdot 3B| = |A^{-1}| \cdot |3B| = \frac{1}{|A|} \cdot 3^3 \cdot |B| = \frac{1}{4} \cdot 27 \, |-2| = -\frac{27}{2}$

c) $|5C_2 + 2C_3 - 3C_1, -2C_2 + C_1, -5C_3 + 4C_1| =$

$|5C_2, -2C_2 + C_1, -5C_3 + 4C_1| + |2C_3, -2C_2 + C_1, -5C_3 + 4C_1| +$

$|-3C_1, -2C_2 + C_1, -5C_3 + 4C_1| =$

$\cancel{|5C_2, -2C_2}, -5C_3 + 4C_1| + |5C_2, C_1, -5C_3 + 4C_1| + |2C_3, -2C_2, -5C_3 + 4C_1| +$

$+ |2C_3, C_1, -5C_3 + 4C_1| + |-3C_1, \cancel{-2C_2}, -5C_3 + 4C_1| + \cancel{|-3C_1, C_1}, -5C_3 + 4C_1| =$

$= |5C_2, C_1, -5C_3| + \cancel{|5C_2, C_1, 4C_1|} + |2C_3, \cancel{-2C_2, -5C_3|} + |2C_3, -2C_2, 4C_1| +$

$+ \cancel{|2C_3, C_1, -5C_3|} + \cancel{|2C_3, C_1, 4C_1|} + |-3C_1, -2C_2, -5C_3| + \cancel{|-3C_1, -2C_2, 4C_1|}$

$= -25|C_2, C_1, C_3| - 16|C_3, C_2, C_1| - 30|C_1, C_2, C_3| =$

$= 25 \cdot |A| + 16|A| - 30|A| = 11|A| = 11 \cdot 4 = 44.$

Escaneado con CamScanner