

TEMA 3 20B

① $\begin{pmatrix} 1 & a & -7 & | & 4a-1 \\ 1 & 1+a & -(a+6) & | & 3a+1 \\ 0 & a & -6 & | & 3a-2 \end{pmatrix}$ $|A| = [6(1+a) - 7a] - [-a(a+6) - 6a] =$
 $= -6 - 6a - 7a - [-a^2 - 6a - 6a] = -6 - 13a + a^2 + 12a =$
 $= a^2 - a - 6 = 0 \rightarrow a = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2} = \begin{matrix} 3 \\ -2 \end{matrix}$

a) Si $a \neq -2, 3$ $\text{rg } A = 3 = \text{rg } A^* = n^{\circ} \text{ inc} \Rightarrow \text{SCD}$

Si $a = -2$ $\begin{pmatrix} 1 & -2 & -7 & | & -9 \\ 1 & -1 & -4 & | & -5 \\ 0 & -2 & -6 & | & -8 \end{pmatrix}$ $\text{rg } A = 2 = \text{rg } A^* < n^{\circ} \text{ inc} \rightarrow \text{SCJ}$
 $|A^*| = \begin{vmatrix} 1 & -2 & -9 \\ 1 & -1 & -5 \\ 0 & -2 & -8 \end{vmatrix} = 8 + 18 - (10 + 16) = 0$

Si $a = 3$ $\begin{pmatrix} 1 & 3 & -7 & | & 11 \\ 1 & 4 & -9 & | & 10 \\ 0 & 3 & -6 & | & 7 \end{pmatrix}$ $\text{rg } A = 2 \neq \text{rg } A^* = 3 \rightarrow \text{SCF}$
 $|A^*| = \begin{vmatrix} 1 & 3 & 11 \\ 1 & 4 & 10 \\ 0 & 3 & 7 \end{vmatrix} = 28 + 33 - 30 - 21 = 10 \neq 0$

b) $\begin{pmatrix} 1 & -2 & | & -9+7\lambda \\ 1 & -1 & | & -5+4\lambda \end{pmatrix} \quad z = \lambda \quad |B| = 1$
 $x = \frac{\begin{vmatrix} -9+7\lambda & -2 \\ -5+4\lambda & -1 \end{vmatrix}}{1} = \frac{9-7\lambda-10+8\lambda}{1} = \lambda-1$
 $y = \frac{\begin{vmatrix} 1 & -9+7\lambda \\ 1 & -5+4\lambda \end{vmatrix}}{1} = \frac{-5+4\lambda+9-7\lambda}{1} = 4-3\lambda$
 $(\lambda-1, 4-3\lambda, \lambda) \quad \forall \lambda \in \mathbb{R}$

c) $a = -3$ $\begin{pmatrix} 1 & -3 & -7 & | & -13 \\ 1 & -2 & -3 & | & -8 \\ 0 & -3 & -6 & | & -11 \end{pmatrix}$ SCD $|A| = (-3)^2 - (-3) - 6 = 9 + 3 - 6 = 6$

$x = \frac{\begin{vmatrix} -13 & -3 & -7 \\ -8 & -2 & -3 \\ -11 & -3 & -6 \end{vmatrix}}{6} = \frac{-8}{6} = -\frac{4}{3}$ $y = \frac{\begin{vmatrix} 1 & -13 & -7 \\ 1 & -8 & -3 \\ 0 & -11 & -6 \end{vmatrix}}{6} = \frac{14}{6} = \frac{7}{3}$

$z = \frac{\begin{vmatrix} 1 & -3 & -13 \\ 1 & -2 & -8 \\ 0 & -3 & -11 \end{vmatrix}}{6} = \frac{4}{6} = \frac{2}{3}$ $(-\frac{4}{3}, \frac{7}{3}, \frac{2}{3})$

② $\begin{pmatrix} a & 2 & 1 & | & 0 \\ a & -1 & 2 & | & 0 \\ 1 & -a & 2 & | & 0 \end{pmatrix}$ $|A| = (-2a - a^2 + 4) - (-1 - 2a^2 + 4a) = -2a - a^2 + 4 + 1 + 2a^2 - 4a =$
 $= a^2 - 6a + 5 = 0 \rightarrow a = \frac{6 \pm \sqrt{36-20}}{2} = \frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2} = \begin{matrix} 5 \\ 1 \end{matrix}$

Si $a \neq 1, 5$ $\text{rg } A = 2 = \text{rg } A^* = n^{\circ} \text{ inc} \Rightarrow \text{SCD} \rightarrow (0, 0, 0)$

Si $a = 1$ $\begin{pmatrix} 1 & 2 & 1 & | & 0 \\ 1 & -1 & 2 & | & 0 \\ 1 & -1 & 2 & | & 0 \end{pmatrix}$ $\text{rg } A = 2 = \text{rg } A^* \rightarrow \text{SCF}$
 $z = \lambda$ $\begin{pmatrix} 1 & 2 & | & -\lambda \\ 1 & -1 & | & -2\lambda \end{pmatrix}$ $|B| = -3$ $x = \frac{\begin{vmatrix} -\lambda & 2 \\ -2\lambda & -1 \end{vmatrix}}{-3} = \frac{\lambda+4\lambda}{-3} = -\frac{5\lambda}{3}$
 $(-\frac{5\lambda}{3}, \frac{\lambda}{3}, \lambda) \quad \forall \lambda \in \mathbb{R}$ $y = \frac{\begin{vmatrix} 1 & -\lambda \\ 1 & -2\lambda \end{vmatrix}}{-3} = \frac{-2\lambda+\lambda}{-3} = \frac{\lambda}{3}$

Si $a = 5$ $\begin{pmatrix} 5 & 2 & 1 & | & 0 \\ 5 & -1 & 2 & | & 0 \\ 1 & -5 & 2 & | & 0 \end{pmatrix}$ $\text{rg } A = 2 = \text{rg } A^* \rightarrow \text{SCF}$
 $z = \lambda$ $\begin{pmatrix} 5 & 2 & | & -\lambda \\ 5 & -1 & | & -2\lambda \end{pmatrix}$ $|B| = -15$ $x = \frac{\begin{vmatrix} -\lambda & 2 \\ -2\lambda & -1 \end{vmatrix}}{-15} = \frac{5\lambda}{-15} = -\frac{\lambda}{3}$
 $(-\frac{\lambda}{3}, \frac{\lambda}{3}, \lambda) \quad \forall \lambda \in \mathbb{R}$ $y = \frac{\begin{vmatrix} 5 & -\lambda \\ 5 & -2\lambda \end{vmatrix}}{-15} = \frac{-5\lambda}{-15} = \frac{\lambda}{3}$