TEMA 2 2º BACHILLERATO A

1. Escribe las propiedades de los determinantes
2. Calcula el siguiente determinante por el método Chio:

$$\begin{vmatrix} -1 & 1 & -2 & 1 & -5 \\ 3 & -1 & 4 & -3 & 0 \\ 1 & 2 & 0 & -2 & 2 \\ 6 & -2 & -2 & -1 & 1 \\ 2 & -2 & 1 & -1 & 0 \end{vmatrix}$$

3. Sea la matriz

$$A = \begin{pmatrix} 1 & m & -1 & 3 \\ m & 1 & 2 & m \\ -6 & 3 & -14 & m \end{pmatrix}$$

Estudia el rango de A en función de los valores de m.

4. Se consideran las matrices A=$\begin{pmatrix} k & 2 & 1 \\ k & 3 & 2 \\ 1 & k & -4 \end{pmatrix}$; B=$\begin{pmatrix} 1 & 1 & 3 \\ 0 & -3 & -2 \\ 2 & 4 & -1 \end{pmatrix}$

   a. Obtener los valores de k para los que A es singular.
   b. Halle, si es posible, la matriz inversa de A en el caso de k=0 y resuelva AX-X=B

5. Sean A y B dos matrices cuadradas de orden 4, cuyos determinantes son $|A| = 6$ y $|B| = -\frac{3}{4}$. Si A=(C_1, C_2, C_3, C_4). Calcular:

a) $|A^t B^4|$ b) $|A^{-1} \cdot 6B|$

c) $|D| = |6C_2 - 3C_4 + 2C_1, \quad -3C_2 + 4C_1, \quad -5C_3 + 4C_1, \quad 7C_4 - C_3|$

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(2)
$$\begin{vmatrix} -1 & 1 & -2 & 1 & -5 \\ 3 & -1 & 4 & -3 & 0 \\ 1 & 2 & 0 & -2 & 2 \\ 6 & -2 & -2 & -1 & 1 \\ 2 & -2 & 1 & -1 & 0 \end{vmatrix} \xrightarrow[F_3 = F_3 - 2F_4]{F_1 = F_1 + 5F_4} \begin{vmatrix} 29 & -9 & -12 & -4 & 0 \\ 3 & -1 & 4 & -3 & 0 \\ -11 & 6 & 4 & 0 & 0 \\ 6 & -2 & -2 & -1 & 1 \\ 2 & -2 & 1 & -1 & 0 \end{vmatrix} \longrightarrow (-1) \begin{vmatrix} 29 & -9 & -12 & -4 \\ 3 & -1 & 4 & -3 \\ -11 & 6 & 4 & 0 \\ 2 & -2 & 1 & -1 \end{vmatrix}$$

$$\xrightarrow[\substack{C_2 = C_2 + 2C_3 \\ C_4 = C_4 + C_3}]{C_1 = C_1 - 2C_3} (-1) \begin{vmatrix} 53 & -33 & -12 & -16 \\ -5 & 7 & 4 & 1 \\ -19 & 14 & 4 & 4 \\ 0 & 0 & 1 & 0 \end{vmatrix} = (-1)(-1) \begin{vmatrix} 53 & -33 & -16 \\ -5 & 7 & 1 \\ -19 & 14 & 4 \end{vmatrix} = (1484 + 1120 + 627) - (2128 +$$

$$+ 742 + 660) = 3231 - 3530 = -299$$

(3) $A = \begin{pmatrix} 1 & m & -1 & 3 \\ m & 1 & 2 & m \\ -6 & 3 & -14 & m \end{pmatrix}$

$$\begin{vmatrix} 1 & m & -1 \\ m & 1 & 2 \\ -6 & 3 & -14 \end{vmatrix} = (-14 - 3m - 12m) - (6 + 6 - 14m^2) = -14 - 15m - 12 + 14m^2 =$$

$$= -14m^2 - 15m - 26 = 0 \longrightarrow m_1 = 2, \quad m_2 = -13/14$$

Si $m \neq 2, m \neq -\frac{13}{14}$ rg $A = 3$

Si $m = 2$ $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ -6 & 3 & 2 \end{vmatrix} = 0$ rg $A = 2$

Si $m = -13/14$ $\begin{vmatrix} 1 & -13/14 & 3 \\ -13/14 & 1 & -13/14 \\ -6 & 3 & -13/14 \end{vmatrix} = \frac{2760}{7} \neq 0 \rightarrow$ rg $A = 3$

(4) $|A| = \begin{vmatrix} k & 2 & 1 \\ k & 3 & 2 \\ 1 & k & -4 \end{vmatrix} = (-12k + k^2 + 4) - (3 + 2k^2 - 8k) = -k^2 - 4k + 1 = 0$

$$k = \frac{4 \pm \sqrt{16+4}}{-2} = \frac{4 \pm \sqrt{20}}{-2} = \frac{4 \pm 2\sqrt{5}}{-2} = -2 \pm \sqrt{5}$$

a) si $k = -2 + \sqrt{5}$, $k = -2 - \sqrt{5} \Rightarrow |A| = 0 \Rightarrow$ A es singular porque $\nexists$ inversa.

b) $AX - X = B \rightarrow (A - I)X = B \rightarrow X = (A - I)^{-1} \cdot B$

$$C = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & -4 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 2 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & -5 \end{pmatrix}$$

$|C| = (10 + 4) - (2) = 12$

$C_{11} = -10$ $\quad C_{21} = +10$ $\quad C_{31} = 2$

$C_{12} = 2$ $\quad C_{22} = 4$ $\quad C_{32} = +2$

$C_{13} = -2$ $\quad C_{23} = +2$ $\quad C_{33} = -2$

$$C^{-1} = \frac{1}{12} \begin{pmatrix} -10 & 10 & 2 \\ 2 & 4 & 2 \\ -2 & 2 & -2 \end{pmatrix}$$

$$X = \frac{1}{12} \begin{pmatrix} -10 & 10 & 2 \\ 2 & 4 & 2 \\ -2 & 2 & -2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 0 & -3 & -2 \\ 2 & 4 & -1 \end{pmatrix} = \frac{1}{12} \begin{pmatrix} -6 & -32 & -52 \\ 6 & -2 & -4 \\ -6 & -16 & -8 \end{pmatrix}$$

$k=0$ $A=\begin{pmatrix} 0 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & -4 \end{pmatrix}$ $|A| = 4-3 = 1$

$A_{11} = -12$ $A_{21} = 8$ $A_{31} = 1$

$A_{12} = +2$ $A_{22} = -1$ $A_{32} = 0$

$A_{13} = -3$ $A_{23} = 2$ $A_{33} = 0$

$A^{-1} = \begin{pmatrix} -12 & 8 & 1 \\ 2 & -1 & 0 \\ -3 & 2 & 0 \end{pmatrix}$

(5) $|A| = 6$, $|B| = -\frac{3}{4}$

a) $|A^t B^4| = |A^t| \cdot |B^4| = |A| \cdot |B|^4 = 6 \cdot \left(-\frac{3}{4}\right)^4 = \frac{6 \cdot 81}{256} = \frac{486}{256} = \frac{243}{128}$

b) $|A^{-1} \cdot 6B| = |A^{-1}| \cdot |6B| = \frac{1}{|A|} \cdot 6^4 \cdot |B| = \frac{1}{6} \cdot 6^4 \cdot \left(-\frac{3}{4}\right) = 6^3 \cdot \left(-\frac{3}{4}\right) = \frac{-216 \cdot 3}{4} =$

$= \frac{-648}{4} = -162$

c) $|D| = |6C_2, -3C_2+4C_1, -5C_3+4C_1, 7C_4-C_3| +$

$+ |-3C_4, -3C_2+4C_1, -5C_3+4C_1, 7C_4-C_3| +$

$+ |2C_1, -3C_2+4C_1, -5C_3+4C_1, 7C_4-C_3| =$

$= |\cancel{6C_2, -3C_2}, -5C_3+4C_1, 7C_4-C_3| + |6C_2, 4C_1, -5C_3+4C_1, 7C_4-C_3| +$

$+ |-3C_4, -3C_2, -5C_3+4C_1, 7C_4-C_3| + |-3C_4, 4C_1, -5C_3+4C_1, 7C_4-C_3| +$

$+ |2C_1, -3C_2, -5C_3+4C_1, 7C_4-C_3| + |\cancel{2C_1, 4C_1}, -5C_3+4C_1+7C_4-C_3| =$

$= |6C_2, 4C_1, -5C_3, 7C_4-C_3| + |6C_2, \cancel{4C_1, 4C_1}, 7C_4-C_3| +$

$+ |-3C_4, -3C_2, -5C_3, 7C_4-C_3| + |-3C_4, -3C_2, 4C_1, 7C_4-C_3| +$

$+ |-3C_4, 4C_1, -5C_3, 7C_4-C_3| + |-3C_4, \cancel{4C_1, 4C_1, 7C_4}-C_3| +$

$+ |2C_1, -3C_2, -5C_3, 7C_4-C_3| + |\cancel{2C_1, -3C_2, 4C_1}, +7C_4-C_3| =$

$= |6C_2, 4C_1, -5C_3, 7C_4| + |6C_2, \cancel{4C_1, -5C_3, -C_3}| +$

$+ |\cancel{-3C_4, -3C_2, -5C_3}, 7C_4| + |-3C_4, \cancel{-3C_2, -5C_3}, -C_3| +$

$+ |\cancel{-3C_4, -3C_2, 4C_1}, 7C_4| + |-3C_4, -3C_2, 4C_1, -C_3| +$

$+ |\cancel{-3C_4, 4C_1, -5C_3}, 7C_4| + |-3C_4, \cancel{4C_1, -5C_3}, -C_3| +$

$+ |2C_1, -3C_2, -5C_3, 7C_4| + |2C_1, \cancel{-3C_2, -5C_3}, -C_3| =$

$= -840\, |C_2, C_1, C_3, C_4| + (-36)\, |C_4, C_2, C_1, C_3| + 210\, |C_1, C_2, C_3, C_4| =$

$= 840\,|A| - 36\,|A| + 210\,|A| = 1014 \cdot |A| = 1014 \cdot 6 = 6084$