TEMA 6: 2º BACHILLERATO A

1. a) Halla el simétrico de A(6,-1,-3) con respecto al plano $\pi: 5x - 4y - z = -7$

b) Halla la distancia del punto A al plano.

2. Halla la ecuación general del plano que corta a los ejes de coordenadas en los puntos (-2,0,0); (0,3,0); (0,0,-4). Halla los puntos de la recta x+2=y-3=z que están a distancia 4 de este plano.

3. Halla la ecuación del plano que contiene a la recta

$$r: \begin{cases} x - z = -3 \\ y - z = 5 \end{cases}$$

Y es perpendicular al plano $\pi: x + 2y - 4z + 7 = 0$. Calcula el ángulo formado por la recta y el plano dados.

4. Dadas las rectas $r: \left\{ \frac{x-2}{3} = \frac{y-1}{-2} = z \right.$ $\qquad s: \left\{ \frac{x+1}{2} = \frac{y+2}{-1} = \frac{z-1}{2} \right.$

Halla la recta perpendicular común a r y s. Halla la distancia mínima entre ellas.

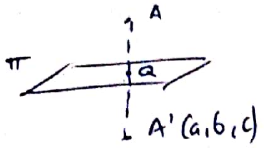
5. Sean las rectas:

$$r: x = \frac{y-1}{-1} = \frac{z-2}{2} \qquad s: \begin{cases} x - 3y - 5 = 0 \\ x - 3z - 8 = 0 \end{cases}$$

Halla la ecuación del plano que contiene a r y es paralelo a s. Calcula la distancia entre el plano y la recta s.

TEMA 6. 2ºA

(1) a) $A(6,-1,-3)$ $\quad \pi: 5x-4y-z=-7$



$\vec{v}_r = \vec{n}(5,-4,-1)$

$$r: \begin{cases} x = 6+5\lambda \\ y = -1-4\lambda \\ z = -3-\lambda \end{cases}$$

$$5(6+5\lambda) - 4(-1-4\lambda) - (-3-\lambda) = -7$$
$$30 + 25\lambda + 4 + 16\lambda + 3 + \lambda = -7$$
$$42\lambda = -44 \rightarrow \lambda = -\frac{44}{42} = \frac{-22}{21}$$

$$Q\left(\frac{16}{21}, \frac{67}{21}, -\frac{41}{21}\right)$$

$$\left(\frac{16}{21}, \frac{67}{21}, -\frac{41}{21}\right) = \left(\frac{6+a}{2}, \frac{-1+b}{2}, \frac{-3+c}{2}\right) \qquad A'\left(\frac{-94}{21}, \frac{155}{21}, \frac{-19}{21}\right)$$

b) $$d(A,\pi) = \frac{|5\cdot 6 - 4\cdot(-1) - (-3) + 7|}{\sqrt{25+16+1}} = \frac{44}{\sqrt{42}} = \frac{44\sqrt{42}}{42} = \frac{22\sqrt{42}}{21}\ u$$

(2) $A(-2,0,0)$; $B(0,3,0)$; $C(0,0,-4)$

a) $\vec{AB}(2,3,0)$
$\vec{AC}(2,0,-4)$

$$\pi: \begin{vmatrix} x+2 & y & z \\ 2 & 3 & 0 \\ 2 & 0 & -4 \end{vmatrix} = 0$$

$$(x+2)(-12) - y(-8) + z(-6) = 0$$
$$-12x + 8y - 6z - 24 = 0$$
$$\boxed{\pi: 6x - 4y + 3z + 12 = 0}$$

b) $x+2 = y-3 = z \Rightarrow r: \begin{cases} x = -2+\lambda \\ y = 3+\lambda \\ z = \lambda \end{cases}$

$R(-2+\lambda, 3+\lambda, \lambda)$

$$d(R,\pi) = \frac{|6(-2+\lambda) - 4(3+\lambda) + 3\lambda + 12|}{\sqrt{36+16+9}} = 4$$

$$\frac{|-12+6\lambda-12-4\lambda+3\lambda+12|}{\sqrt{61}} = 4 \rightarrow |5\lambda - 12| = 4\sqrt{61}$$

$$5\lambda - 12 = 4\sqrt{61} \rightarrow \lambda_1 = \frac{4\sqrt{61}+12}{5} \Rightarrow P_1\left(\frac{4\sqrt{61}+2}{5}, \frac{4\sqrt{61}+27}{5}, \frac{4\sqrt{61}+12}{5}\right)$$

$$5\lambda - 12 = -4\sqrt{61} \rightarrow \lambda_2 = \frac{-4\sqrt{61}+12}{5} \Rightarrow P_2\left(\frac{-4\sqrt{61}+2}{5}, \frac{-4\sqrt{61}+27}{5}, \frac{-4\sqrt{61}+12}{5}\right)$$

(3) $\pi_1 \begin{cases} \vec{v_r} \\ R \\ \vec{n} \end{cases}$ $\quad r: \begin{cases} x - z = -3 \\ y - z = 5 \end{cases}$ $\quad \vec{v_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = (1,1,1)$

$\vec{n}(1,2,-4) \qquad R(-3,5,0)$

$$\pi: \begin{vmatrix} x+3 & y-5 & z \\ 1 & 1 & 1 \\ 1 & 2 & -4 \end{vmatrix} = 0 \Rightarrow (x+3)(-6) - (y-5)(-5) + z \cdot 1 = 0$$

$$-6x + 5y + z - 43 = 0$$

$$\pi: 6x - 5y - z + 43 = 0$$

• Ángulo $\cos\beta = \dfrac{|(1,1,1)(1,2,-4)|}{\sqrt{3} \cdot \sqrt{21}} = \dfrac{1}{\sqrt{63}} \qquad \beta = 82{,}77°$

$\alpha = 90° - 82{,}77° = 7{,}23°$

(4) $r: \dfrac{x-2}{3} = \dfrac{y-1}{-2} = z \qquad \vec{v_r}(3,-2,1) \qquad R(2,1,0) \qquad \vec{RS}(-3,-3,1)$

$s: \dfrac{x+1}{2} = \dfrac{y+2}{-1} = \dfrac{z-1}{2} \qquad \vec{v_s}(2,-1,2) \qquad S(-1,-2,1)$

$$\vec{v_r} \times \vec{v_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 1 \\ 2 & -1 & 2 \end{vmatrix} = (-3,-4,1)$$

$$\pi_1: \begin{vmatrix} x-2 & y-1 & z \\ 3 & -2 & 1 \\ -3 & -4 & 1 \end{vmatrix} = 0 \Rightarrow (x-2) \cdot 2 - (y-1)6 + z(-18) = 0$$

$$\pi_1: 2x - 6y - 18z + 2 = 0$$

$$\pi_2: \begin{vmatrix} x+1 & y+2 & z-1 \\ 2 & -1 & 2 \\ -3 & -4 & 1 \end{vmatrix} = 0 \rightarrow (x+1)7 - (y+2)8 + (z-1)(-11) = 0$$

$$\pi_2: 7x - 8y - 11z + 2 = 0$$

$$t: \begin{cases} 2x - 6y - 18z + 2 = 0 \\ 7x - 8y - 11z + 2 = 0 \end{cases}$$

Posición relativa

$\vec{v_r} \times \vec{v_s} \neq 0$

$$[\vec{RS}, \vec{v_r}, \vec{v_s}] = \begin{vmatrix} -3 & -3 & 1 \\ 3 & -2 & 1 \\ 2 & -1 & 2 \end{vmatrix} = (12-3-6) - (-4+3-18) = 3+19 = 22 \neq 0$$

$\Rightarrow$ SE CRUZAN

$$d(r,s) = \frac{[\vec{RS}, \vec{v_r}, \vec{v_s}]}{|\vec{v_r} \times \vec{v_s}|} = \frac{22}{\sqrt{9+16+1}} = \frac{22}{\sqrt{26}} = \frac{11\sqrt{26}}{13} = 4{,}31 \text{ u}$$

(5) $r: x = \dfrac{y-1}{-1} = \dfrac{z-2}{2} \qquad s: \begin{cases} x - 3y - 5 = 0 \\ x - 3z - 8 = 0 \end{cases} \qquad \vec{v_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 0 \\ 1 & 0 & -3 \end{vmatrix} = (9,3,3)$

$\vec{v_r}(1,-1,2)$

$R(0,1,2)$

$\vec{v_s}(9,3,3)$

$S(0,-\frac{5}{3},-\frac{8}{3})$

$$\pi: \begin{vmatrix} x & y-1 & z-2 \\ 1 & -1 & 2 \\ 9 & 3 & 3 \end{vmatrix} = 0 \rightarrow x(-9) - (y-1)(-15) + (z-2)12 = 0$$

$$-9x + 15y + 12z - 39 = 0$$

$$\pi: 3x - 5y - 4z + 13 = 0$$

$\vec{v_s} \cdot \vec{n} = (9,3,3)(3,-5,-4) = 0 \Rightarrow$ Paralelas

$$d(s,\pi) = d(S,\pi) = \frac{|3 \cdot 0 - 5 \cdot (-5/3) - 4 \cdot (-8/3) + 13|}{\sqrt{9+25+16}} = \frac{32}{\sqrt{50}} = \frac{16\sqrt{50}}{25} \text{ u.}$$