I TEMA 1. 2º BACHILLERATO A

1. Dadas las matrices $A = \begin{pmatrix} -3 & 1 & -1 \\ 0 & -1 & 5 \\ 0 & 1 & -4 \end{pmatrix}$ y $B = \begin{pmatrix} 6 & 1 & -1 \\ 0 & -3 & 0 \\ 5 & 0 & -3 \end{pmatrix}$.

   Resuelve el sistema matricial $\begin{cases} 4X - 7Y = A \\ 5X + 2Y = B \end{cases}$

2. Calcula los valores de a, b y c para que la matriz A sea antisimétrica $A = \begin{pmatrix} b & a & 0 \\ a^2 & 0 & c \\ 0 & 2 & b \end{pmatrix}$

3. Calcula A^n, A^3, A^{10} *siendo* $A = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ 0 & -1 & -1 \end{pmatrix}$

4. Dadas las matrices $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2 & 3 \\ 4 & 1 & -2 \end{pmatrix}, B = \begin{pmatrix} -1 & 1 & 3 \\ 1 & 2 & -4 \\ 1 & -5 & 0 \end{pmatrix}$.

   Resuelve la ecuación siguiente: $XA + A^{-1} = B + 2A^t$

5. Estudia según el valor de k el rango de la siguiente matriz

$$\begin{pmatrix} 1 & -2 & 1 \\ 3 & 2 & a \\ 0 & 4 & -1 \end{pmatrix}$$

   Decide para qué valores la matriz es singular.

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1) $A = \begin{pmatrix} -3 & 1 & -1 \\ 0 & -1 & 5 \\ 0 & 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 6 & 1 & -1 \\ 0 & -3 & 0 \\ 5 & 0 & -3 \end{pmatrix}$

$$\left.\begin{matrix} 4X - 7Y = A \\ 5X + 2Y = B \end{matrix}\right\} \Rightarrow \begin{matrix} \cdot(5) \\ \cdot(-4) \end{matrix} \begin{array}{l} \cancel{20X} - 35Y = 5A \\ \underline{-\cancel{20X} - 8Y = -4B} \\ -43Y = 5A - 4B \end{array} \Rightarrow \begin{matrix} \cdot(2) \\ \cdot(7) \end{matrix} \begin{array}{l} 8X - \cancel{14Y} = 2A \\ \underline{35X + \cancel{14Y} = 7B} \\ 43X = 2A + 7B \end{array}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$Y = \frac{5A - 4B}{-43} \qquad X = \frac{2A + 7B}{43}$$

$$X = \frac{\begin{pmatrix} -6 & 2 & -2 \\ 0 & -2 & 10 \\ 0 & 2 & -8 \end{pmatrix} + \begin{pmatrix} 42 & 7 & -7 \\ 0 & -21 & 0 \\ 35 & 0 & -21 \end{pmatrix}}{43} = \begin{pmatrix} 36/43 & 9/43 & -9/43 \\ 0 & -23/43 & 10/43 \\ 35/43 & 2/43 & -29/43 \end{pmatrix}$$

$$Y = \frac{\begin{pmatrix} -15 & 5 & -5 \\ 0 & -5 & 25 \\ 0 & 5 & -20 \end{pmatrix} - \begin{pmatrix} 24 & 4 & -4 \\ 0 & -12 & 0 \\ 20 & 0 & -12 \end{pmatrix}}{-43} = \begin{pmatrix} 39/43 & -1/43 & 1/43 \\ 0 & -7/43 & -25/43 \\ 20/43 & -5/43 & 8/43 \end{pmatrix}$$

2) A es antisimétrica si $-A = A^t$

$$-A = \begin{pmatrix} -b & -a & 0 \\ -a^2 & 0 & -c \\ 0 & -2 & -b \end{pmatrix}$$

$$A^t = \begin{pmatrix} b & a^2 & 0 \\ a & 0 & 2 \\ 0 & c & b \end{pmatrix}$$

$$-A = A^t \Rightarrow \begin{pmatrix} -b & -a & 0 \\ -a^2 & 0 & -c \\ 0 & -2 & -b \end{pmatrix} = \begin{pmatrix} b & a^2 & 0 \\ a & 0 & 2 \\ 0 & c & b \end{pmatrix}$$

$$\left.\begin{matrix} -b = b \\ -a = a^2 \\ -a^2 = a \\ -c = 2 \\ c = -2 \end{matrix}\right\} \begin{array}{l} b = 0 \\ a^2 = -a \Rightarrow a^2 + a = 0 \Rightarrow a(a+1) = 0 \begin{cases} a = 0 \\ a = -1 \end{cases} \\ c = -2 \end{array}$$

Hay dos soluciones

$$A_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 2 & 0 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -2 \\ 0 & 2 & 0 \end{pmatrix}$$

$a = 0$ $\quad$ $a = -1$

$b = 0$ $\quad$ $b = 0$

$c = -2$ $\quad$ $c = -2$

③

$$A^2 = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ 0 & -1 & -1 \end{pmatrix}\begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 2 \\ 1 & 1 & -1 \\ -1 & -1 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} -1 & 0 & 2 \\ 1 & 1 & -1 \\ -1 & -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$A^n = \begin{cases} n = 3k \longrightarrow I \\ n = 3k+1 \longrightarrow A \\ n = 3k+2 \longrightarrow A^2 \end{cases} \qquad A^3 = I \qquad A^{10} = A^{3\cdot 3+1} = A$$

④ $XA + A^{-1} = B + 2A^t \Rightarrow X = (B + 2A^t - A^{-1})\, A^{-1}$

$$A^{-1} \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 3 & 0 & 1 & 0 \\ 4 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \xrightarrow[4F_1 - F_3]{} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 3 & 0 & 1 & 0 \\ 0 & 3 & 2 & 4 & 0 & -1 \end{array}\right) \xrightarrow[\substack{2F_1 + F_2 \\ 3F_2 + 2F_3}]{}$$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 3 & 2 & 1 & 0 \\ 0 & -2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 13 & 8 & 3 & -2 \end{array}\right) \xrightarrow[\substack{13F_1 - 3F_3 \\ 13F_2 - 3F_3}]{} \left(\begin{array}{ccc|ccc} 26 & 0 & 0 & 2 & 4 & 6 \\ 0 & -26 & 0 & -24 & 4 & 6 \\ 0 & 0 & 13 & 8 & 3 & -2 \end{array}\right) \xrightarrow{\substack{F_1/26 \\ F_2/-26 \\ F_3/13}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2/26 & 4/26 & 6/2\ldots \\ 0 & 1 & 0 & 24/26 & -4/26 & -6/\ldots \\ 0 & 0 & 1 & 8/13 & 3/13 & -2/\ldots \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 1/13 & 2/13 & 3/13 \\ 12/13 & -2/13 & -3/13 \\ 8/13 & 3/13 & -2/13 \end{pmatrix}$$

$$X = \left[\begin{pmatrix} -1 & 1 & 3 \\ 1 & 2 & -4 \\ 1 & -5 & 0 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 & 4 \\ 1 & -2 & 1 \\ 0 & 3 & 2 \end{pmatrix} - \begin{pmatrix} 1/13 & 2/13 & 3/13 \\ 12/13 & -2/13 & -3/13 \\ 8/13 & 3/13 & -2/13 \end{pmatrix}\right] \cdot \begin{pmatrix} 1/13 & 2/13 & 3/13 \\ 12/13 & -2/13 & -3/13 \\ 8/13 & 3/13 & -2/13 \end{pmatrix} =$$

$$= \begin{pmatrix} 12/13 & 1/13 & 140/13 \\ 27/13 & -24/13 & -27/13 \\ 5/13 & 10/13 & -30/13 \end{pmatrix}\begin{pmatrix} 1/13 & 2/13 & 3/13 \\ 12/13 & -2/13 & -3/13 \\ 8/13 & 3/13 & -2/13 \end{pmatrix} = \begin{pmatrix} 1264/169 & 422/169 & -277/169 \\ -445/169 & 33/169 & 199/169 \\ -275/169 & -160/169 & 85/169 \end{pmatrix}$$

⑤ $$\begin{pmatrix} 1 & -2 & 1 \\ 3 & 2 & a \\ 0 & 4 & -1 \end{pmatrix} \xrightarrow[3F_1 - F_2]{} \begin{pmatrix} 1 & -2 & 1 \\ 0 & -8 & 3-a \\ 0 & 4 & -1 \end{pmatrix} \xrightarrow[F_2 + 2F_3]{} \begin{pmatrix} 1 & -2 & 1 \\ 0 & -8 & 3-a \\ 0 & 0 & 1-a \end{pmatrix}$$

$1 - a = 0 \Rightarrow a = 1$

Si $a \neq 1 \Rightarrow rg\, A = 3$

Si $a = 1 \Rightarrow rg\, A = 2$

A es singular, si no tiene inversa, es decir, si $rg\, A \neq$ orden $A = 3$

Luego A es singular si $a = 1$