

TEMA 2 2º BACHILLERATO A

1. Escribe las propiedades de los determinantes
2. Calcula el siguiente determinante:

$$\begin{vmatrix} -1 & 1 & -2 & 1 & -6 \\ 3 & -1 & 4 & -3 & 0 \\ 1 & -3 & 0 & -2 & 2 \\ 6 & -2 & -2 & -1 & 1 \\ 2 & -2 & 1 & -1 & 0 \end{vmatrix}$$

3. Sea la matriz

$$A = \begin{pmatrix} 1 & m & -1 & 3 \\ m & 1 & 2 & m \\ -6 & 3 & -14 & m \end{pmatrix}$$

Estudia el rango de A en función de los valores de m.

4. Se consideran las matrices $A = \begin{pmatrix} k & 2 & 1 \\ k & 3 & 2 \\ 1 & k & -4 \end{pmatrix}$; $B = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -2 \\ 2 & 4 & 1 \end{pmatrix}$

- a. Obtener los valores de k para los que A es singular.
- b. Halle, si es posible, la matriz inversa de A en el caso de $k=0$ y resuelva $AX-X=B$

5. Sean A y B dos matrices cuadradas de orden 3, cuyos determinantes son $|A| = 4$ y $|B| = -2$. Si $A = (C_1, C_2, C_3)$. Calcular:

a) $|A^t B^4|$

b) $|A^{-1} \cdot 3B|$

c) $|D| = |5C_2 + 2C_3 - 3C_1, -2C_2 + C_1, -5C_3 + 4C_1|$

TEMA 2

$$\begin{aligned}
 & \textcircled{2} \begin{vmatrix} -1 & 1 & -2 & 1 & -6 \\ 3 & -1 & 4 & -3 & 0 \\ 1 & -3 & 0 & -2 & 2 \\ 6 & -2 & -2 & -1 & \textcircled{1} \\ 2 & -2 & 1 & -1 & 0 \end{vmatrix} \begin{array}{l} \\ \\ \\ F_1 = F_1 + 6F_4 \\ F_3 = F_3 - 2F_4 \end{array} = \begin{vmatrix} -35 & -11 & -14 & -5 & 0 \\ 3 & -1 & 4 & -3 & 0 \\ -11 & 1 & 4 & 0 & 0 \\ \hline 6 & -2 & -2 & -1 & \textcircled{1} \\ 2 & -2 & 1 & -1 & 0 \end{vmatrix} = \\
 & - \begin{vmatrix} 35 & -11 & -14 & -5 \\ 3 & -1 & 4 & -3 \\ -11 & 1 & 4 & 0 \\ 2 & -2 & \textcircled{1} & -1 \end{vmatrix} \begin{array}{l} \\ \\ C_1 = C_1 - 2C_3 \\ C_2 = C_2 + 2C_3 \\ C_4 = C_4 + C_3 \end{array} = \begin{vmatrix} 63 & -39 & -14 & -19 \\ -5 & 7 & 4 & 1 \\ -14 & 9 & 4 & 4 \\ \hline 0 & 0 & \textcircled{1} & 0 \end{vmatrix} = \begin{vmatrix} 63 & -39 & -19 \\ -5 & 7 & 1 \\ -19 & 9 & 4 \end{vmatrix} = \\
 & = 3360 - (3874) = -514
 \end{aligned}$$

$$\textcircled{3} \quad A = \begin{pmatrix} 1 & m & -1 & 3 \\ m & 1 & 2 & m \\ -6 & 3 & -14 & m \end{pmatrix}$$

$$\begin{vmatrix} 1 & m & -1 \\ m & 1 & 2 \\ -6 & 3 & -14 \end{vmatrix} = -14 - 3m - 12m - 6 - 6 + 14m^2 = 14m^2 - 15m - 26 = 0$$

$$m = \frac{15 \pm 41}{28} \begin{cases} m_1 = 2 \\ m_2 = -\frac{26}{28} = -\frac{13}{14} \end{cases} \quad m = (-15/8)$$

Si $m \neq 2, -13/14 \Rightarrow \text{rg } A = 3$

$$\text{Si } m = 2 \quad \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 1 & 2 & 2 \\ -6 & 3 & -14 & 2 \end{pmatrix} \quad \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & 2 \\ 3 & -14 & 2 \end{vmatrix} = 0$$

$\text{rg } A = 2$

$$\text{Si } m = -13/14 \quad \begin{pmatrix} 1 & -13/14 & -1 & 3 \\ -13/14 & 1 & 2 & -13/14 \\ -6 & 3 & -14 & -13/14 \end{pmatrix} \quad \begin{vmatrix} -13/14 & -1 & 3 \\ 1 & 2 & -13/14 \\ 3 & -14 & -13/14 \end{vmatrix} = -15138$$

$\text{rg } A = 3$

Si $m \neq 2$ $\text{rg } A = 3$, si $m = 2$ $\text{rg } A = 2$

(4) $A = \begin{pmatrix} k & 2 & 1 \\ k & 3 & 2 \\ 1 & k & -4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -2 \\ 2 & 4 & 1 \end{pmatrix}$

a) $\begin{vmatrix} k & 2 & 1 \\ k & 3 & 2 \\ 1 & k & -4 \end{vmatrix} = -12k + k^2 + 4 - 3 - 2k^2 + 8k = -k^2 - 4k + 1 = 0$

Si $k = \frac{4 \pm \sqrt{20}}{-2}$ $\xrightarrow{k = -2 \pm \sqrt{5}}$ la matriz es singular.

b) $k=0 \quad A = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & -4 \end{pmatrix} \quad |A| = 1$

$A_{11} = -12 \quad A_{21} = +8 \quad A_{31} = 1$

$A_{12} = +2 \quad A_{22} = -1 \quad A_{32} = 0$

$A_{13} = -3 \quad A_{23} = +2 \quad A_{33} = 0$

$A^{-1} = \begin{pmatrix} -12 & 8 & 1 \\ 2 & -1 & 0 \\ -3 & 2 & 0 \end{pmatrix}$

$AX - X = B \Rightarrow (A - I)X = B \Rightarrow X = (A - I)^{-1} B$

$C = A - I = \begin{pmatrix} -1 & 2 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & -5 \end{pmatrix} \quad |C| = 14 - 2 = 12$

$C_{11} = -10 \quad C_{21} = 10 \quad C_{31} = 2$

$C_{12} = 2 \quad C_{22} = 4 \quad C_{32} = +2$

$C_{13} = -2 \quad C_{23} = +2 \quad C_{33} = -2$

$C^{-1} = \frac{1}{12} \begin{pmatrix} -10 & 10 & 2 \\ 2 & 4 & 2 \\ -2 & 2 & -2 \end{pmatrix}$

$X = \frac{1}{12} \begin{pmatrix} -10 & 10 & 2 \\ 2 & 4 & 2 \\ -2 & 2 & -2 \end{pmatrix} \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -2 \\ 2 & 4 & 1 \end{pmatrix} = \frac{1}{12} \begin{pmatrix} -6 & 38 & -48 \\ 6 & 14 & 0 \\ -6 & -2 & -12 \end{pmatrix} = \begin{pmatrix} -1/2 & 19/6 & -4 \\ 1/2 & 7/6 & 0 \\ -1/2 & -1/6 & 1 \end{pmatrix}$

(5) a) $|A^t B^4| = |A^t| \cdot |B^4| = |A| \cdot |B|^4 = 4 \cdot (-2)^4 = 4 \cdot 16 = 64$

b) $|A^{-1} \cdot 3B| = |A^{-1}| \cdot |3B| = \frac{1}{|A|} \cdot 3^3 \cdot |B| = \frac{1}{4} \cdot 27 \cdot (-2) = -\frac{27}{2}$

$$c) |5C_2 + 2C_3 - 3C_1, -2C_2 + C_1, -5C_3 + 4C_1| =$$

$$|5C_2, -2C_2 + C_1, -5C_3 + 4C_1| + |2C_3, -2C_2 + C_1, -5C_3 + 4C_1| +$$

$$|-3C_1, -2C_2 + C_1, -5C_3 + 4C_1| =$$

$$|5C_2, -2C_2, -5C_3 + 4C_1| + |5C_2, C_1, -5C_3 + 4C_1| + |2C_3, -2C_2, -5C_3 + 4C_1| +$$

$$+ |2C_3, C_1, -5C_3 + 4C_1| + ~~| -3C_1, -2C_2, -5C_3 + 4C_1 |~~ + ~~| -3C_1, C_1, -5C_3 + 4C_1 |~~$$

$$= |5C_2, C_1, -5C_3| + ~~|5C_2, C_1, 4C_1|~~ + ~~|2C_3, -2C_2, -5C_3|~~ + |2C_3, -2C_2, 4C_1| +$$

$$+ ~~|2C_3, C_1, -5C_3|~~ + ~~|2C_3, C_1, 4C_1|~~ + ~~| -3C_1, -2C_2, -5C_3 |~~ + ~~| -3C_1, -2C_2, 4C_1 |~~$$

$$= -25 |C_2, C_1, C_3| - 16 |C_3, C_2, C_1| - 30 |C_1, C_2, C_3| =$$

$$= 25 \cdot |A| + 16 |A| - 30 |A| = 11 |A| = 11 \cdot 4 = 44.$$