

③ Sean $A = \begin{pmatrix} 4 & 8 \\ 7 & 11 \end{pmatrix}$ $B = \begin{pmatrix} 10 & 1 \\ 8 & 18 \end{pmatrix}$. Resuelve el sistema $\begin{cases} 2X + 3Y = A \\ 5X - 2Y = 2B \end{cases}$

$$\begin{cases} 2X + 3Y = A \\ 5X - 2Y = 2B \end{cases} \rightarrow \begin{cases} \cdot(5) & 10X + 15Y = 5A \\ \cdot(-2) & -10X + 4Y = -4B \end{cases}$$

$$\underline{19Y = 5A - 4B} \rightarrow Y = \frac{5A - 4B}{19}$$

$$\downarrow$$

$$\begin{cases} \cdot(2) & 4X + 6Y = 2A \\ \cdot(3) & 15X - 6Y = 6B \end{cases} \rightarrow X = \frac{2A + 6B}{19}$$

$$\underline{19X = 2A + 6B}$$

$$X = \frac{1}{19} \left[\begin{pmatrix} 8 & 16 \\ 14 & 22 \end{pmatrix} + \begin{pmatrix} 60 & 6 \\ 48 & 108 \end{pmatrix} \right] = \frac{1}{19} \begin{pmatrix} 68 & 22 \\ 62 & 130 \end{pmatrix} = \begin{pmatrix} 68/19 & 22/19 \\ 62/19 & 130/19 \end{pmatrix}$$

$$Y = \frac{1}{19} \left[\begin{pmatrix} 20 & 40 \\ 35 & 55 \end{pmatrix} - \begin{pmatrix} 40 & 4 \\ 32 & 72 \end{pmatrix} \right] = \frac{1}{19} \begin{pmatrix} -20 & 36 \\ 3 & -17 \end{pmatrix} = \begin{pmatrix} -20/19 & 36/19 \\ 3/19 & -17/19 \end{pmatrix}$$

④ $AX = B \cdot A \rightarrow X = A^{-1} \cdot B \cdot A$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{F_1 + F_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{F_2 \leftrightarrow F_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array} \right) \xrightarrow{F_1 + F_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array} \right)$$

$$A^{-1} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 0 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ -1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 4 \\ 1 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ -1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 1 \\ 1 & 3 & -1 \\ -1 & 2 & 2 \end{pmatrix}$$

⑤ $A^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$A^3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^n = \begin{cases} n = 4k \rightarrow I \\ n = 4k+1 \rightarrow A \\ n = 4k+2 \rightarrow A^2 \\ n = 4k+3 \rightarrow A^3 \end{cases}$$

$$A^{26} = A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$