TEMA **1º BACHILLERATO A** (1)

1. a) Pasar a forma polar y trigonométrica: $-4+\sqrt{2}\,i$
   b) Pasar a forma binómica y trigonométrica: $\sqrt{5}_{120^\circ}$
   c) Pasar a forma binómica y polar z= 4(cos 30º+i sen 30º) (1,5 puntos)
2. Calcular m y n para que se cumpla:
   $\frac{m+2i}{1-ni} = \sqrt{2}_{30^\circ}$ (1,5 puntos)

3. Resuelve la ecuación $z^9 = 4+3i$ (1,5 puntos)
4. Calcula:

   $a)\ \left(\frac{-1+3i}{3-2i}\right)^{12} =$ b) $\frac{(5_{55}\cdot 2_{65})^4}{7_{120^\circ}} =$ (1,5 puntos)

5. Halla dos números complejos tales que su diferencia sea un número real, la parte real de la suma sea 4 y su producto sea -1+8i (1,5 puntos)
6. Calcula:

   $a)\ \frac{i^9-i^4+i^{12}-i^{21}}{1-i^5} =$ b) $\frac{2i^5-3i^3+(-2i)^7}{6i^6+i^{15}} =$ (1,5 puntos)

7. Resuelve (3-2i)(-5+4i)+6z= z(-1-4i)-7i (1 punto)

TEMA 5: 1ºA (1)

(1) a) $z=-4+\sqrt{2}i$ (1,5)

$r=\sqrt{(-4)^2+(\sqrt{2})^2}=\sqrt{18}$

$\alpha=\text{arc tg }\frac{\sqrt{2}}{-4}=160{,}53^\circ$

Forma polar $z=\sqrt{18}_{160{,}53^\circ}$

Forma trigonométrica $z=\sqrt{18}(\cos 160{,}53^\circ+i\text{ sen }160{,}53^\circ)$

b) $z=\sqrt{5}_{120^\circ}$

$a=\sqrt{5}\cos 120^\circ=-\frac{\sqrt{5}}{2}$

$b=\sqrt{5}\text{ sen }120^\circ=\frac{\sqrt{15}}{2}$

Forma binómica $z=-\frac{\sqrt{5}}{2}+\frac{\sqrt{15}}{2}i$

Forma trigonométrica $z=\sqrt{5}(\cos 120^\circ+i\text{ sen }120^\circ)$

c) $z=4(\cos 30^\circ+i\text{ sen }30^\circ)$

$a=4\cos 30^\circ=2\sqrt{3}$

$b=4\text{ sen }30^\circ=2$

Forma binómica $z=2\sqrt{3}+2i$

Forma polar $z=4_{30^\circ}$

(2) (1,5) $\frac{m+2i}{1-ni}=\sqrt{2}_{30^\circ}$

$\sqrt{2}_{30^\circ}=\sqrt{2}\cos 30^\circ+\sqrt{2}\text{ sen }30^\circ i=\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i$

$\frac{m+2i}{1-ni}=\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i \longrightarrow m+2i=(1-ni)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i\right).$

$m+2i=\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i-\frac{\sqrt{6}}{2}ni-\frac{\sqrt{2}}{2}ni^2$

$m+2i=\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}n\right)+\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{6}}{2}n\right)i$

Luego $\left.\begin{array}{l} m=\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}n \\ 2=\frac{\sqrt{2}}{2}-\frac{\sqrt{6}}{2}n \end{array}\right\}$ $n=\frac{2-\sqrt{2}/2}{-\sqrt{6}/2}=-\frac{2\sqrt{6}+\sqrt{3}}{3}=-1{,}06$

$m=\frac{2\sqrt{6}-2\sqrt{3}}{3}=0{,}48$

(3) (1,5) $z^9=4+3i$

$r=\sqrt{4^2+3^2}=\sqrt{25}=5$

$\alpha=\text{arc tg }\frac{3}{4}=36{,}87^\circ$

$s=\sqrt[9]{5}$

$\beta\rightarrow$ $36{,}87^\circ:9=4{,}1$; $360^\circ:9=40^\circ$

Soluciones: $\sqrt[9]{5}_{4{,}1^\circ}$, $\sqrt[9]{5}_{44{,}1^\circ}$, $\sqrt[9]{5}_{84{,}1^\circ}$, $\sqrt[9]{5}_{124{,}1^\circ}$, $\sqrt[9]{5}_{164{,}1^\circ}$

$\sqrt[9]{5}_{204{,}1^\circ}$, $\sqrt[9]{5}_{244{,}1^\circ}$, $\sqrt[9]{5}_{284{,}1^\circ}$, $\sqrt[9]{5}_{324{,}1^\circ}$

(4) a) $\left(\frac{-1+3i}{3-2i}\right)^{12} =$
(1,5)

$$\frac{-1+3i}{3-2i} = \frac{(-1+3i)(3+2i)}{(3-2i)(3+2i)} = \frac{-3-2i+9i+6i^2}{9-4i^2} = \frac{-9+7i}{13} = -\frac{9}{13}+\frac{7}{13}i$$

$$r = \sqrt{\left(-\frac{9}{13}\right)^2+\left(\frac{7}{13}\right)^2} = \frac{\sqrt{130}}{13}$$

$$\alpha = \operatorname{arc\,tg} \frac{7/13}{-9/13} = 142{,}13^\circ$$

$$\left(\frac{-1+3i}{3-2i}\right)^{12} = \left(\frac{\sqrt{130}}{13}\,{}_{142{,}13^\circ}\right)^{12} = \left(\frac{\sqrt{130}}{13}\right)^{12}_{1705{,}56^\circ} = \left(\frac{\sqrt{130}}{13}\right)^{12}_{265{,}56^\circ}$$

b) $$\left(\frac{5_{55^\circ}\cdot 2_{65^\circ}}{7_{120^\circ}}\right)^4 = \left(\frac{10_{120^\circ}}{7_{120^\circ}}\right)^4 = \left(\frac{10}{7}{}_{0^\circ}\right)^4 = \left(\frac{10}{7}\right)^4_{0^\circ}$$

(5) $z_1 = a+bi$ $\quad z_1 - z_2 = (a+bi)-(c+di) = (a-c)+(b-d)i$ Sea real
(1,5) $z_2 = c+di$ $\quad \Rightarrow \boxed{(b-d)=0}$

$z_1+z_2 = (a+bi)+(c+di) = (a+c)+(b+d)i$

La parte real sea 4 $\rightarrow \boxed{a+c=4}$

$z_1\cdot z_2 = (a+bi)(c+di) = ac+adi+bci+bdi^2 =$
$= (ac-bd)+(ad+bc)i = -1+8i$

Luego $\boxed{\begin{array}{l} ac-bd=-1 \\ ad+bc=8 \end{array}}$

$b-d=0 \rightarrow b=d$

$a+c=4 \rightarrow a=4-c$

$\left.\begin{array}{l} ac-bd=-1 \\ ad+bc=8 \end{array}\right\} \left\{\begin{array}{l} (4-c)c-d^2=-1 \\ (4-c)d+dc=8 \end{array}\right\} \begin{array}{l} 4c-c^2-d^2=-1 \\ 4d-\cancel{cd}+\cancel{dc}=8 \rightarrow d=\frac{8}{4}=2 \;\boxed{d=2} \end{array}$

$4c-c^2-4=-1 \rightarrow 4c-c^2=3 \rightarrow c^2-4c+3=0$

$c = \frac{4\pm\sqrt{16-12}}{2} = \frac{4\pm\sqrt{4}}{-2} = \frac{4\pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$

$b=d \rightarrow \boxed{b=2}$ $\quad \boxed{c_1=3}$ $\quad \boxed{c_2=1}$

$a=4-c \rightarrow a_1=4-3=1$
$\quad \hookrightarrow a_2=4-1=3$

Los números son $z_1 = 1+2i$, $z_2 = 3+2i$
$z_1 = 3+2i$, $z_2 = 1+2i$

(6) a) $\dfrac{i^9 - i^4 + i^{12} - i^{21}}{1 - i^5} = \dfrac{i - 1 + 1 - i}{1 - i} = \dfrac{0}{1-i} = 0$

(1,5)

b) $\dfrac{2i^5 - 3i^3 + (-2i)^7}{6i^6 + i^{15}} = \dfrac{2i - 3i^3 - 128\, i^3}{6i^2 + i^3} = \dfrac{2i + 3i + 128\, i}{-6 - i} = \dfrac{133i}{-6-i} =$

$= \dfrac{133\, i\,(-6+i)}{(-6-i)(-6+i)} = \dfrac{-798i + 133i^2}{36 - i^2} = \dfrac{-133 - 798i}{37}$

(7) $(3 - 2i)(-5 + 4i) + 6z = z(-1 - 4i) - 7i$

$-15 + 12i + 10i - 8i^2 + 6z = z(-1 - 4i) - 7i$

$6z - z(-1 - 4i) = -7i + 15 - 12i - 10i - 8$

$z(6 + 1 + 4i) = 7 - 29i$

$z(7 + 4i) = 7 - 29i$

$z = \dfrac{7 - 29i}{7 + 4i} = \dfrac{(7-29i)(7-4i)}{(7+4i)(7-4i)} = \dfrac{49 - 28i - 203i + 116i^2}{49 - 16i^2} = \dfrac{-67 - 231i}{65}$

$= -\dfrac{67}{65} - \dfrac{231}{65}i$