

0,75 (1) a) $A(5, -2, -4) \quad \pi: 6x - 3y + 2z = -7$

$\vec{n} = (6, -3, 2) = \vec{v}_r$

$$r: \begin{cases} x = 5 + 6\lambda \\ y = -2 - 3\lambda \\ z = -4 + 2\lambda \end{cases}$$

$$\begin{aligned} 6(5+6\lambda) - 3(-2-3\lambda) + 2(-4+2\lambda) + 7 &= 0 \\ 36\lambda + 9\lambda + 4\lambda + 30 + 6 - 8 + 7 &= 0 \\ 49\lambda + 35 &= 0 \rightarrow \lambda = -\frac{35}{49} \end{aligned}$$

$$Q\left(\frac{5}{7}, \frac{1}{7}, -\frac{38}{7}\right) \quad \left(\frac{5}{7}, \frac{1}{7}, -\frac{38}{7}\right) = \left(\frac{x+5}{2}, \frac{y-2}{2}, \frac{z-4}{2}\right)$$

$$A' \left(-\frac{25}{7}, \frac{16}{7}, -\frac{48}{7}\right)$$

0,25 b) $r: \frac{x+1}{2} = \frac{y+2}{-1} = \frac{z-1}{2} \quad \vec{v}_r(2, -1, 2) = \vec{n}$

$$2x - y + 2z + D = 0 \Rightarrow 2 \cdot 5 - (-2) + 2 \cdot (-4) + D = 0 \rightarrow D = -4$$

$$\pi: 2x - y + 2z - 4 = 0$$

$$r: \begin{cases} x = -1 + 2\lambda \\ y = -2 - \lambda \\ z = 1 + 2\lambda \end{cases}$$

$$\begin{aligned} 2(-1+2\lambda) - (-2-\lambda) + 2(1+2\lambda) - 4 &= 0 \\ 4\lambda + \lambda + 4\lambda - 2 + 2 + 2 - 4 &= 0 \\ 9\lambda - 2 &= 0 \quad \lambda = \frac{2}{9} \end{aligned}$$

$$Q\left(-\frac{5}{9}, -\frac{20}{9}, \frac{13}{9}\right) \quad \left(-\frac{5}{9}, -\frac{20}{9}, \frac{13}{9}\right) = \left(\frac{x+1}{2}, \frac{y-2}{2}, \frac{z-1}{2}\right)$$

$$A' \left(-\frac{55}{9}, -\frac{22}{9}, \frac{62}{9}\right)$$

0,5 c) $(-1, 5, 2) = \left(\frac{x+5}{2}, \frac{y-2}{2}, \frac{z-4}{2}\right) \quad A' = (-7, 12, 8)$

(2) $A(5, 0, 0), B(0, -6, 0), C(0, 0, 4) \quad \vec{AB}(-5, -6, 0) \quad \vec{AC}(-5, 0, 4)$

0,5 $\pi: \begin{vmatrix} x-5 & y & z \\ -5 & -6 & 0 \\ -5 & 0 & 4 \end{vmatrix} = 0 \quad \begin{aligned} (x-5)(-24) - y(-20) + z(-30) &= 0 \\ -24x + 20y - 30z + 120 &= 0 \end{aligned}$

$$\pi: 12x - 10y + 15z - 60 = 0$$

1,5 $r: x+2 = y-3 = z = \begin{cases} x = -2 + \lambda \\ y = 3 + \lambda \\ z = \lambda \end{cases}$

$$d(P, \pi) = 8$$

$$P_1(14,89; 19,89; 16,89)$$

$$P_2(-5,49; -0,49; -3,49)$$

$$\frac{|12(-2+\lambda) - 10(3+\lambda) + 15\lambda - 60|}{\sqrt{144+100+225}} = 8$$

$$\frac{|-24+12\lambda-30-10\lambda+15\lambda-60|}{\sqrt{469}} = 8 \Rightarrow \frac{|17\lambda-114|}{\sqrt{469}} = 8$$

$$17\lambda - 114 = 8\sqrt{469}$$

$$17\lambda - 114 = -8\sqrt{469}$$

$$\lambda = \dots + 16,89$$

$$\lambda = -3,49$$

$$(3) r: \begin{cases} 2x+3z=-5 \\ y-2z=4 \end{cases} \quad \left| \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 3 \\ 0 & 1 & -2 \end{array} \right| = (-3, 4, 2) = \vec{v}_r \quad R\left(-\frac{5}{2}, 4, 0\right)$$

$$1 \quad \pi: 4x+3y-3z+5=0 \quad \vec{n}(4, 3, -3)$$

$$\left| \begin{array}{ccc} x+\frac{5}{2} & y-4 & z \\ -3 & 4 & 2 \\ 4 & 3 & -3 \end{array} \right| = 0 \quad \left(x+\frac{5}{2}\right)(-18) - (y-4)(-1) + z(-25) = 0$$

$$\pi: -18x - y - 25z - 41 = 0$$

$$\pi: 18x + y + 25z + 41 = 0$$

$$1 \quad \omega \beta = \frac{|(-3, 4, 2) \cdot (4, 3, -3)|}{\sqrt{9+16+4} \sqrt{16+9+9}} = \frac{|-6|}{\sqrt{29} \cdot \sqrt{34}} = \frac{6}{\sqrt{986}} \quad \beta = 78,98^\circ$$

$$\alpha = 11,02^\circ$$

$$(4) r: \begin{cases} \frac{x-3}{5} = \frac{y+2}{-3} = z \end{cases}$$

$$\vec{v}_r(5, -3, 1)$$

$$R(3, -2, 0)$$

$$s: \begin{cases} \frac{x+4}{2} = \frac{y-3}{-2} = \frac{z+1}{2} \end{cases}$$

$$\vec{v}_s(2, -2, 2)$$

$$S(-4, 3, -1)$$

$$\left| \begin{array}{ccc} -7 & 5 & -1 \\ 5 & -3 & 1 \\ 2 & -2 & 2 \end{array} \right| = -8 \neq 0 \quad \text{se cauză}$$

$$\vec{v}_r \times \vec{v}_s = \left| \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 5 & -3 & 1 \\ 2 & -2 & 2 \end{array} \right| = (-4, -8, -4)$$

$$\pi_1: \left| \begin{array}{ccc} x-3 & y+2 & z \\ -4 & -8 & -4 \\ 5 & -3 & 1 \end{array} \right| = 0$$

$$(x-3)(-20) - (y+2)16 + z \cdot 52 = 0$$

$$-20x - 16y + 52z + 28 = 0$$

$$\pi_1: 5x + 4y - 13z - 7 = 0$$

1,5

$$\pi_2: \left| \begin{array}{ccc} x+4 & y-3 & z+1 \\ 2 & -2 & 2 \\ -4 & -8 & -4 \end{array} \right| = 0$$

$$(x+4)24 - (y-3) \cdot 0 + (z+1)(-24) = 0$$

$$24x - 24z + 72 = 0$$

$$\pi_2: x - z + 3 = 0$$

$$t: \begin{cases} 5x + 4y - 13z - 7 = 0 \\ x - z + 3 = 0 \end{cases}$$

0,5

$$d(r, s) = \frac{|R\vec{S}, \vec{v}_r, \vec{v}_s|}{|\vec{v}_r \times \vec{v}_s|} = \frac{|1-81|}{\sqrt{16+64+16}} = \frac{8}{\sqrt{96}} = \frac{8}{4\sqrt{6}} = \frac{2}{\sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3} u$$

$$(5) r: 2x = \frac{y-3}{-1} = \frac{z+5}{-2} \Rightarrow \frac{x}{\frac{1}{2}} = \frac{y-3}{-1} = \frac{z+5}{-2} \quad \vec{v}_r\left(\frac{1}{2}, -1, -2\right) \quad R(0, 3, -5)$$

$$s: \begin{cases} 5x-2y-4=0 \\ x-3z-6=0 \end{cases}$$

$$\left| \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 5 & -2 & 0 \\ 1 & 0 & -3 \end{array} \right| = (6, 15, 2) = \vec{v}_s$$

$$S(0, -2, -2)$$

$$\left| \begin{array}{ccc} x & y-3 & z+5 \\ \frac{1}{2} & -1 & -2 \\ 6 & 15 & 2 \end{array} \right| = x \cdot 28 - (y-3)13 + (z+5) \frac{27}{2} = 0$$

$$28x - 13y + \frac{27}{2}z + \frac{213}{2} = 0$$

$$\pi: 56x - 26y + 27z + 213 = 0$$

0,5

$$\vec{n} \cdot \vec{v}_s = (56, -26, 27) \cdot (6, 15, 2) = 0 \quad S \notin \pi \quad \text{Paralel}$$

0,5

$$d(S, \pi) = \frac{|211|}{\sqrt{56^2 + 26^2 + 27^2}} = \frac{211}{\sqrt{4541}} = 3,13 u$$