

2) a)  $|C| = |-3C_2 + 4C_3 - C_1, C_3, 7C_2 - C_1| =$   
 $| -3C_2, C_3, 7C_2 - C_1 | + | 4C_3, C_3, 7C_2 - C_1 | + | -C_1, C_3, 7C_2 - C_1 | =$   
 $| -3C_2, C_3, 7C_2 | + | -3C_2, C_3, -C_1 | + | -C_1, C_3, 7C_2 | + | -C_1, C_3, -C_1 | =$   
 $3|C_2, C_3, C_1| - 7|C_1, C_3, C_2| = 3|A| + 7|A| = 10|A| = 10 \cdot 8 = 80$

b)  $|B^t \cdot GA| = |B^t| |GA| = |B| \cdot 6^3 |A| = -\frac{1}{3} \cdot 216 \cdot 8 = -576$

c)  $|A^{-1} \cdot B^3| = |A^{-1}| \cdot |B|^3 = \frac{1}{|A|} \cdot |B|^3 = \frac{1}{8} \cdot \left(-\frac{1}{3}\right)^3 = -\frac{1}{216}$

3) a)  $\begin{vmatrix} a & -1 & 4 \\ 3 & a & 0 \\ -1 & 0 & 1 \end{vmatrix} = a^2 + 4a + 3 = 0 \quad \begin{cases} a = -1 \\ a = -3 \end{cases}$

Si  $a \neq -1, -3$  es regular.

b) Si  $a = 1$

$AX + B = 2X \Rightarrow AX - 2X = -B \Rightarrow (A - 2I)X = -B \Rightarrow X = (A - 2I)^{-1}(-B)$

$A - 2I = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -1 & 4 \\ 3 & -1 & 0 \\ -1 & 0 & -1 \end{pmatrix}$

$|C| = -1 - (4 + 3) = -8$

$A_{11} = 1 \quad A_{21} = -1 \quad A_{31} = 4$   
 $A_{12} = -(-3) = 3 \quad A_{22} = 5 \quad A_{32} = -(-12) = 12$   
 $A_{13} = -1 \quad A_{23} = -(-1) = 1 \quad A_{33} = 4$

$C^{-1} = -\frac{1}{8} \begin{pmatrix} 1 & -1 & 4 \\ 3 & 5 & 12 \\ -1 & 1 & 4 \end{pmatrix}$

$X = -\frac{1}{8} \begin{pmatrix} 1 & -1 & 4 \\ 3 & 5 & 12 \\ -1 & 1 & 4 \end{pmatrix} \begin{pmatrix} -2 & -1 & 0 \\ 0 & 1 & -1 \\ -3 & -1 & 1 \end{pmatrix} = -\frac{1}{8} \begin{pmatrix} -14 & -6 & 5 \\ -42 & -10 & 7 \\ -10 & -2 & 3 \end{pmatrix}$

4)  $|A| = \begin{vmatrix} 1 & -2 & 1 \\ 1 & m & 3 \\ 5 & -1 & m \end{vmatrix} = (m^2 - 1 - 30) - (5m + 3 - 2m) = m^2 - 3m - 28 = 0 \quad \begin{cases} m = 7 \\ m = -4 \end{cases}$

Si  $m \neq 7, -4$   $\text{rg } A = 3$

Si  $m = 7$   $\text{rg } A = 2$

Si  $m = -4$   $\text{rg } A = 2$

5)  $\begin{vmatrix} -2 & 0 & 4 & -1 \\ 3 & -1 & 0 & 2 \\ 5 & -1 & 1 & 2 \\ 2 & 0 & 3 & 0 \end{vmatrix} \xrightarrow{\substack{F_1 - 4F_3 \\ F_4 - 3F_3}} \begin{vmatrix} -22 & 4 & 0 & -9 \\ 3 & -1 & 0 & 2 \\ 5 & -1 & 1 & 2 \\ -13 & 3 & 0 & -6 \end{vmatrix} \rightarrow \begin{vmatrix} -22 & 4 & -9 \\ 3 & -1 & 2 \\ -13 & 3 & -6 \end{vmatrix} =$

$(-132 - 81 - 104) - (-117 - 132 - 72) = -317 - (-321) = 4$