## TEMAS 8 Y 9. 2º BACHILLERATO A

1. Calcula el valor de la derivada en el punto (-5,-2) de la siguiente función:
$3x^2y^3 - 6x^4y + 5y^5x^4 - 7 = 0$ (1 punto)

2. Calcula los siguientes límites:
   a. $\lim_{x\to 1} \frac{1-\cos(x-1)}{(lnx)^2}$
   b. $\lim_{x\to 0} (6x^2 - 5x)^{sen\, 3x}$
   c. $\lim_{x\to 0} x^{\frac{1}{\ln x}}$
   d. Calcula los valores de a para que el $\lim_{x\to +\infty} \frac{2x}{\ln(e^{ax}-1)} = 4$ (3 puntos)

3. Calcula las siguientes derivadas:
   a. $y = \left(\frac{\cos(2x+1)}{x^2+x-2}\right)^{x^3}$
   b. $y = \sqrt[(6x^2-7x)]{\frac{\ln(3x-2)}{e^{sen(4x+5)}}}$ (2 puntos)

4. Calcula el valor de a y b para que la función f(x) sea continua y derivable en x=0

$$f(x) = \begin{cases} (x+a)e^{-bx} & si \quad x < 0 \\ ax^2 + bx + 1 & si \quad x \geq 0 \end{cases}$$ ( 2 puntos)

5. Sea la función $f(x) = 3x^4 - 6x^2 + 1$, calcula la monotonía, la curvatura, los máximos y mínimos, y los puntos de inflexión. Y la recta tangente y la recta normal a la curva en el punto x=0 (2 puntos)

TEMAS 8,9 2°A

① $3x^2y^3 - 6x^4y + 5y^5x^4 - 7 = 0$ $P(-5,-2)$

(1) $6xy^3 + 9x^2y^2y' - 24x^3y - 6x^4y' + 25y^4y'x^4 + 20y^5x^3 = 0$

$y'(9x^2y^2 - 6x^4 + 25y^4x^4) = -6xy^3 + 24x^3y - 20y^5x^3$

$$y' = \frac{-6xy^3 + 24x^3y - 20y^5x^3}{9x^2y^2 - 6x^4 + 25y^4x^4}$$

$$y'(-5,-2) = \frac{-74240}{247150} = -\frac{7424}{24715} = -0{,}300384382$$

② (3) a) $\lim_{x\to 1} \frac{1-\cos(x-1)}{(\ln x)^2} = \left[\frac{0}{0}\right] \underset{LH}{=} \lim_{x\to 1} \frac{\operatorname{sen}(x-1)}{2\ln x \cdot \frac{1}{x}} = \left[\frac{0}{0}\right] \underset{LH}{=} \lim_{x\to 1} \frac{\cos(x-1)}{2\frac{1}{x}\cdot\frac{1}{x} + 2\ln x\left(-\frac{1}{x^2}\right)} = \frac{1}{2+0} =$

b) $\lim_{x\to 0} (6x^2-5x)^{\operatorname{sen} 3x} = [0^0]$

$\ln\left(\lim_{x\to 0} (6x^2-5x)^{\operatorname{sen}3x}\right) = \lim_{x\to 0}\left(\ln(6x^2-5x)^{\operatorname{sen}3x}\right) = \lim_{x\to 0} \operatorname{sen}3x \cdot \ln(6x^2-5x)$

$[0\cdot\infty] = \lim_{x\to 0} \frac{\ln(6x^2-5x)}{\frac{1}{\operatorname{sen}3x}} = \left[\frac{\infty}{\infty}\right] \underset{LH}{=} \lim_{x\to 0} \frac{\frac{12x-5}{6x^2-5x}}{-(\operatorname{sen}3x)^{-2}\cdot\cos 3x\cdot 3} = \lim_{x\to 0} \frac{(12x-5)(\operatorname{sen}3x)^2}{-(6x^2-5x)\cdot 3\cos 3x} = \left[\frac{0}{0}\right]$

$= \lim_{x\to 0} \frac{12(\operatorname{sen}3x)^2 + (12x-5)\cdot 2\cdot\operatorname{sen}3x\cdot\cos 3x\cdot 3}{-(12x-5)\cdot 3\cos 3x + (-1(6x^2-5x)\cdot 9(-\operatorname{sen}3x)} = \frac{0}{15+0} = \frac{0}{15} = 0$

$\lim_{x\to 0} (6x^2-5x)^{\operatorname{sen} x} = e^0$

c) $\lim_{x\to 0} x^{\frac{1}{\ln x}} = [0^\infty]$

$\ln\left(\lim_{x\to 0} x^{\frac{1}{\ln x}}\right) = \lim_{x\to 0}\left(\ln x^{1/\ln x}\right) = \lim_{x\to 0}\left[\frac{1}{\ln x}\cdot\ln x\right] = \lim_{x\to 0} 1 = 1$

$\lim_{x\to 0} x^{\frac{1}{\ln x}} = e^1 = e$

d) $\lim_{x\to +\infty} \frac{2x}{\ln(e^{ax}-1)} = 4$

$\lim_{x\to+\infty} \frac{2x}{\ln(e^{ax}-1)} = \left[\frac{\infty}{\infty}\right] \underset{LH}{=} \lim_{x\to+\infty} \frac{2}{\frac{ae^{ax}}{e^{ax}-1}} = \lim_{x\to+\infty} \frac{2(e^{ax}-1)}{ae^{ax}} = \left[\frac{\infty}{\infty}\right] \underset{LH}{=}$

$= \lim_{x\to\infty} \frac{2a\cancel{e^{ax}}}{a^2\cancel{e^{ax}}} = \lim_{x\to\infty} \frac{2a}{a^2} = \lim_{x\to\infty} \frac{2a}{a^2} = 4$ $\rightarrow 4a^2 = 2a$

$4a^2 - 2a = 0$

$\Rightarrow \boxed{a = \frac{1}{2}}$

$\Rightarrow \boxed{a = 0}$

(3) a) $y=\left(\frac{\cos(2x+1)}{x^2+x-2}\right)^{x^3} \rightarrow \ln y = \ln\left(\frac{\cos(2x+1)}{x^2+x-2}\right)^{x^3} \Rightarrow \ln y = x^3 \cdot \ln\left(\frac{\cos(2x+1)}{x^2+x-2}\right)$

(2)

$$\frac{y'}{y} = 3x^2 \cdot \ln\left(\frac{\cos(2x+1)}{x^2+x-2}\right) + x^3 \cdot \frac{\frac{-2\,\mathrm{sen}(2x+1)\cdot(x^2+x-2) - \cos(2x+1)\cdot(2x+1)}{(x^2+x-2)^{\cancel{2}}}}{\frac{\cos(2x+1)}{\cancel{x^2+x-2}}}$$

$$y' = \left[3x^2 \cdot \ln\left(\frac{\cos(2x+1)}{x^2+x-2}\right) + x^3 \cdot \frac{-2\,\mathrm{sen}(2x+1)(x^2+x-2) - \cos(2x+1)\cdot(2x+1)}{\cos(2x+1)\,(x^2+x-2)}\right] \cdot \left[\frac{\cos(2x+1)}{x^2+x-2}\right]^{x^3}$$

b) $y = \sqrt[(6x^2-7x)]{\frac{\ln(3x-2)}{e^{\mathrm{sen}(4x+5)}}} = \left[\frac{\ln(3x-2)}{e^{\mathrm{sen}(4x+5)}}\right]^{1/6x^2-7x}$

$\Rightarrow \ln y = \frac{1}{6x^2-7x} \cdot \ln\left[\frac{\ln(3x-2)}{e^{\mathrm{sen}(4x+5)}}\right]$

$$\frac{y'}{y} = \left[\frac{-(12x-7)}{(6x^2-7x)^2} \cdot \ln\left(\frac{\ln(3x-2)}{e^{\mathrm{sen}(4x+5)}}\right) + \frac{1}{6x^2-7x} \cdot \frac{\frac{\frac{3}{3x-2}\,e^{\mathrm{sen}(4x+5)} - \ln(3x-2)\cdot 4\cos(4x+5)\,e^{\mathrm{sen}(4x+5)}}{[e^{\mathrm{sen}(4x+5)}]^{\cancel{2}}}}{\frac{\ln(3x-2)}{\cancel{e^{\mathrm{sen}(4x+5)}}}}\right]$$

$$y' = \left[\frac{-(12x-7)}{(6x^2-7x)^2} \cdot \ln\left(\frac{\ln(3x-2)}{e^{\mathrm{sen}(4x+5)}}\right) + \frac{1}{6x^2-7x} \cdot \frac{\frac{3}{3x-2}\,e^{\mathrm{sen}(4x+5)} - \ln(3x+2)\,4\cdot\cos(4x-5)\cdot e^{\mathrm{sen}(4x+5)}}{\ln(3x-2)\cdot e^{\mathrm{sen}(4x+5)}}\right] \cdot \sqrt[6x^2-7x]{\frac{\ln(3x-2)}{e^{\mathrm{sen}(4x+5)}}}$$

(4)

(2)

$f(x) = \begin{cases} (x+a)e^{-bx} & \text{si } x<0 \quad \text{Cont. combinación pol y exp.} \\ ax^2+bx+1 & \text{si } x \geq 0 \quad \text{Cont. polinómica} \end{cases} \Rightarrow f'(x) = \begin{cases} e^{-bx}+(x+a)(-b)e^{-bx} & \text{si } x<0 \\ 2ax+b & \text{si } x\geq 0 \end{cases}$

En $x=0$

$f(0)=1$

$\lim_{x\to 0^-} (x+a)e^{-bx} = a e^0 = a$ $\quad \boxed{a=1}$

$\lim_{x\to 0^+} (ax^2+bx+1) = 1$ //

$f'(0^-) = e^0 + (a)(-b)e^0 = 1-ab$

$f'(0^+) = 2a\cdot 0 + b = b$ //

$\boxed{1-ab=b}$

$\left.\begin{array}{l} a=1 \\ 1-ab=b \end{array}\right\}$ $\boxed{a=1}$ $\quad 1-b=b \rightarrow 2b=1 \rightarrow \boxed{b=\frac{1}{2}}$

Si $a=1$, $b=1/2$ la función es continua y derivable en $\mathbb{R}$

(5) $f(x) = 3x^4 - 6x^2 + 1$

(2) $f'(x) = 12x^3 - 12x = 0 \Rightarrow 12x(x^2-1)=0 \rightarrow x=0;\ x=1;\ x=-1$

$(-\infty,-1)$ $f'<0$ Dec $>$ Mín $(-1,-2)$

$(-1,0)$ $f'>0$ crec $>$ Máx $(0,1)$

$(0,1)$ $f'<0$ Dec $>$ Mín $(1,-2)$

$(1,+\infty)$ $f'>0$ crec

$f''(x) = 36x^2 - 12 = 0 \rightarrow x = \sqrt{\frac{12}{36}} = \pm\sqrt{\frac{1}{3}}$

$(-\infty, -\sqrt{1/3})$ $f''>0$ $\cup$ Cóncava $>$ PI $(-\sqrt{1/3}, -1/3)$

$(-\sqrt{1/3}, +\sqrt{1/3})$ $f''<0$ $\cap$ Convexa $>$ PI $(\sqrt{1/3}, -1/3)$

$(\sqrt{1/3}, +\infty)$ $f''>0$ $\cup$ cóncava

En $x=0$ $\quad f'(0)=0$

$y_0 = f(0) = 1$

RT. $(y-1) = 0(x-0) \rightarrow y=1$

RN $(y-1) = -\frac{1}{0}(x-0) \Rightarrow \cancel{\exists}$