(2)

TEMA . 1º BACHILLERATO A

1. a) Pasar a forma polar y trigonométrica: $-6+\sqrt{7}\,i$

   b) Pasar a forma binómica y trigonométrica: $\sqrt{8}_{150º}$

   c) Pasar a forma binómica y polar z= 9(cos 20º+i sen 20º) (1,5 puntos)

2. Calcular m y n para que se cumpla:

   $\frac{m-3i}{1-ni} = \sqrt{3}_{60º}$ (1,5 puntos)

3. Resuelve la ecuación $z^9 = -3+7i$ (1,5 puntos)

4. Calcula:

   a) $\left(\frac{-1+2i}{5-4i}\right)^{14} =$ b) $\frac{(3_{75}\cdot 2_{85})^5}{5_{180º}} =$ (1,5 puntos)

5. Halla dos números complejos sabiendo que el producto es -8 y que uno de ellos es el cuadrado del otro. (1,5 puntos)

6. Calcula:

   a) $\frac{i^{10}-i^5+i^{13}-i^{22}}{1-i^6} =$ b) $\frac{2i^6-3i^4+(-2i)^5}{5i^7+i^{14}} =$ (1,5 puntos)

7. Resuelve (4-2i)(-4+3i)+7z= z(-2-3i)-8i (1 punto)

TEMA 5. 1ºA (2)

① a) $z=-6+\sqrt{7}i$

(1,5)

$r=\sqrt{(-6)^2+(\sqrt{7})^2}=\sqrt{85}$

$\alpha=\text{arc tg}\,\frac{\sqrt{7}}{-6}=156,20°$

F. polar: $z=\sqrt{85}_{156,20°}$

F. Trigonométrica $z=\sqrt{85}\cos 156,20°+i\sqrt{85}\,\text{sen}\,156,2°$

b) $z=\sqrt{8}_{150°}$

$a=\sqrt{8}\cos 150°=-\sqrt{6}$

$b=\sqrt{8}\,\text{sen}\,150°=\sqrt{2}$

F. Binómica $z=-\sqrt{6}+\sqrt{2}i$

F. Trigonométrica $z=\sqrt{8}\cos 150°+\sqrt{8}\,\text{sen}\,150°i$

c) $z=9(\cos 20°+i\,\text{sen}\,20°)$

$a=9\cos 20°=8,46$

$b=9\,\text{sen}\,20°=3,08$

F. Binómica: $z=8,46+3,08i$

F. Polar $z=9_{20°}$

② $\frac{m-3i}{1-ni}=\sqrt{3}_{60°}$

(1,5)

$\sqrt{3}_{60°}=\sqrt{3}\cos 60°+\sqrt{3}\,\text{sen}\,60°i=\frac{\sqrt{3}}{2}+\frac{3}{2}i$

$\frac{m-3i}{1-ni}=\left(\frac{\sqrt{3}}{2}+\frac{3}{2}i\right)\rightarrow m-3i=\left(\frac{\sqrt{3}}{2}+\frac{3}{2}i\right)(1-ni)=\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}ni+\frac{3}{2}i-\frac{3}{2}ni^2=$

$m-3i=\left(\frac{\sqrt{3}}{2}+\frac{3}{2}n\right)+\left(-\frac{\sqrt{3}}{2}n+\frac{3}{2}\right)$

$\Rightarrow\ m=\frac{\sqrt{3}}{2}+\frac{3}{2}n$

$-3=-\frac{\sqrt{3}}{2}n+\frac{3}{2}$

$n=\frac{-3-3/2}{-\sqrt{3}/2}=3\sqrt{3}=5,2$

$m=\frac{\sqrt{3}}{2}+\frac{3}{2}3\sqrt{3}=5\sqrt{3}=8,66$

③ $z^9=-3+7i$

(1,5)

$r=\sqrt{(-3)^2+7^2}=\sqrt{58}$

$\alpha=\text{arc tg}\,\frac{7}{-3}=113,20°$

$\longrightarrow\ s=\sqrt[9]{\sqrt{58}}=\sqrt[18]{58}$

$\beta\Rightarrow\ 113,20°:9=12,58°$

$360°:9=40°$

Soluciones: $\sqrt[18]{58}_{12,58°}$, $\sqrt[18]{58}_{52,58°}$, $\sqrt[18]{58}_{92,58°}$, $\sqrt[18]{58}_{132,58°}$,

$\sqrt[18]{58}_{172,58°}$, $\sqrt[18]{58}_{212,58°}$, $\sqrt[18]{58}_{252,58°}$, $\sqrt[18]{58}_{292,58°}$

$\sqrt[18]{58}_{332,58°}$

④ a) $\left(\frac{-1+2i}{5-4i}\right)^{14}$

$$\left(\frac{-1+2i}{5-4i}\right) = \frac{(-1+2i)(5+4i)}{(5-4i)(5+4i)} = \frac{-5-4i+10i+8i^2}{25-16i^2} = \frac{-13+6i}{41} = -\frac{13}{41}+\frac{6}{41}i$$

$$r = \sqrt{\left(-\frac{13}{41}\right)^2+\left(\frac{6}{41}\right)^2} = \frac{\sqrt{205}}{41}$$

$$\alpha = \text{arc tg} \frac{6/41}{-13/41} = 155{,}22$$

$$\left(\frac{-1+2i}{5-4i}\right)^{14} = \left(\frac{\sqrt{205}}{41}{}_{155{,}22^\circ}\right)^{14} = \left(\frac{\sqrt{205}}{41}\right)^{14}_{2173{,}08^\circ} = \left(\frac{\sqrt{205}}{41}\right)^{14}_{13{,}08^\circ}$$

b) $$\left(\frac{(3_{75^\circ}\cdot 2_{85})^5}{5_{180^\circ}}\right) = \frac{(6_{160^\circ})^5}{5_{180}} = \frac{7776_{800^\circ}}{5_{180^\circ}} = \left(\frac{7776}{5}\right)_{620^\circ} = \left(\frac{7776}{5}\right)_{260^\circ}$$

⑤ $z_1 \cdot z_2 = -8$

$z_1 = z_2^2$

$z_1 = a+bi$

$z_2 = c+di$

$$z_1\cdot z_2 = (a+bi)\cdot(c+di) = ac+adi+bci+bdi^2 = (ac-bd)+(ad+bc)i = -8$$

$$a+bi = (c+di)^2 = c^2+2cdi+d^2i^2 = (c^2-d^2)+2cdi$$

$$\left.\begin{array}{l} ac-bd=-8 \\ ad+bc=0 \\ a=c^2-d^2 \\ b=2cd \end{array}\right\}$$

$ad+2cd\cdot c=0 \rightarrow ad+2c^2d=0$

$d(a+2c^2)=0$ → $\boxed{d=0}$, $\boxed{a+2c^2=0}$

* Si $d=0 \rightarrow b=0$

$\rightarrow a^2=c^2$

$\rightarrow ac=-8$

$$\left\{ a=-\frac{8}{c} \right.$$

$$\left(-\frac{8}{c}\right)^2 = c^2 \rightarrow \frac{64}{c^2}=c^2 \rightarrow c^4=64 \rightarrow c=\pm\sqrt[4]{2^6}$$

$$c=\pm\sqrt{2^3}=\pm\sqrt{8}$$

Los números son $z_1 = -\frac{8}{\sqrt{8}}+0i$, $z_2=\sqrt{8}+0i$ No vale

$z_2 = \frac{8}{\sqrt{8}}+0i$, $z_2=-\sqrt{8}+0i$ ✓

* Si $a+2c^2=0 \rightarrow a=-2c^2$

$-2c^2=c^2-d^2 \Rightarrow d^2=3c^2$

$-2c^3-2cdd=-8$

$$\left.\begin{array}{l} d^2=3c^2 \\ -2c^3-2cd^2=-8 \end{array}\right\}$$

$$\left.\begin{array}{l} d^2=3c^2 \\ c^3+cd^2=4 \end{array}\right\}$$

$c^3+c\cdot 3c^2=4$

$4c^2=4 \rightarrow c=\pm 1$

Si $c_1=1 \rightarrow a_1=-2 \rightarrow -2=1-d^2 \rightarrow d=\pm\sqrt{3} \rightarrow b=\pm\sqrt{3}$

Los números son $z_1=-2+\sqrt{3}i$, $z_2=1+\sqrt{3}i$

$z_1=-2-\sqrt{3}i$, $z_2=1-\sqrt{3}i$

Si $c_2 = -1 \rightarrow a_2 = -2 \rightarrow -2 = 1 - d^2 \rightarrow d = \pm\sqrt{3} \rightarrow b = \pm\sqrt{3}$

Los números son $z_1 = -2 + \sqrt{3}i \quad , \quad z_2 = -1 + \sqrt{3}i$

$z_1 = -2 - \sqrt{3}i \quad , \quad z_2 = 1 + \sqrt{3}i$

6 a) $\dfrac{i^{10} - i^5 + i^{13} - i^{22}}{1 - i^6} = \dfrac{i^2 - i + i - i^2}{1 - i^2} = \dfrac{-1 - i + i + 1}{1 + 1} = \dfrac{0}{2}$

b) $\dfrac{2i^6 - 3i^4 + (-2i)^5}{5i^7 + i^{14}} = \dfrac{2i^2 - 3 + (-2)^5 i^5}{5i^3 + i^2} = \dfrac{-2 - 3 - 32i}{-5i - 1} =$

$= \dfrac{(-5 - 32i)(-1 + 5i)}{(-1 - 5i)(-1 + 5i)} = \dfrac{5 - 25i + 32i - 160i^2}{1 - 25i^2} = \dfrac{165 + 7i}{1 + 25} = \dfrac{165}{26} + \dfrac{7}{26}i$

7 $(4 - 2i)(-4 + 3i) + 7z = z(-2 - 3i) - 8i$

$-16 + 12i + 8i - 6i^2 + 7z = z(-2 - 3i) - 8i$

$7z - z(-2 - 3i) = -8i + 16 - 12i - 8i + 6i^2$

$7z + 2z + 3zi = -28i + 10$

$z(5 + 3i) = 10 - 28i \longrightarrow z = \dfrac{10 - 28i}{5 + 3i} = \dfrac{(10 - 28i)(5 - 3i)}{(5 + 3i)(5 - 3i)} =$

$z = \dfrac{50 - 30i - 140i + 84i^2}{25 - 9i^2} = \dfrac{-34 - 170i}{34} = -\dfrac{34}{34} - \dfrac{170}{34}i = -1 - 5i$