TEMA DERIVADAS. 1º BACHILLERATO B

1. Calcula la derivada quinta de
   a. $f(x) = 7x^7 - 3x^5 - 5x^4 - 2$
   b. $f(x) = cos\ (2x)$
2. Calcula las siguientes derivadas:
   a. $f(x) = \ sen^5(arc\ cos(\sqrt{5x^2 - 6x}))$
   b. $f(x) = 7^{x^3+6x-5}\ ln\ (\mathrm{tg}(8x^2 + 2x))$
   c. $f(x) = \frac{arc\ \cos(e^{7x^2+6x-1})\cdot sen(6x^2-x)}{(\log_5(3x-5)}$
   d. $f(x) = \frac{cos(4x^2+5x)\ln\ (3^{x^4})}{sen(log_7(x^2-2))}$
   e. $f(x) = ln^6(arc\ tg\sqrt[5]{2x+5})$
3. Halla los valores a y b para que la función sea continua y derivable:

$$f(x) = \begin{cases} x^2 + ax - 5 & si\ x < 2 \\ \dfrac{1}{x-1} + b & si\ x \geq 2 \end{cases}$$

4. Estudia la continuidad y derivabilidad de la siguiente función

$$f(x) = \begin{cases} \dfrac{1}{x+3} & si\ \ x \leq 0 \\ -x^2 + 14 & si\ \ 0 < x \leq\ 3 \\ \sqrt{x^2+16} & si\ x > 3 \end{cases}$$

TEMA DERIVADAS 1ºB

① a) $f(x) = 7x^7 - 3x^5 - 5x^4 - 2$

$f'(x) = 49x^6 - 15x^4 - 20x^3$

$f''(x) = 294x^5 - 60x^3 - 60x^2$

$f'''(x) = 1470x^4 - 180x^2 - 120x$

$f^{IV}(x) = 5880x^3 - 360x - 120$

$f^{V}(x) = 17640x^2 - 360$

b) $f(x) = \cos(2x)$

$f'(x) = -2\,\text{sen}(2x)$

$f''(x) = -4\cos(2x)$

$f'''(x) = 8\,\text{sen}(2x)$

$f^{IV}(x) = 16\cos(2x)$

$f^{V}(x) = -32\,\text{sen}(2x)$

② a) $f(x) = \text{sen}^5\left(\text{arc}\cos\left(\sqrt{5x^2-6x}\right)\right)$

$f'(x) = 5\,\text{sen}^4\left(\text{arc}\cos\sqrt{5x^2-6x}\right) \cdot \cos\left(\text{arc}\cos\sqrt{5x^2-6x}\right) \cdot \dfrac{-1}{\sqrt{1-\left(\sqrt{5x^2-6x}\right)^2}} \cdot \dfrac{1}{2}(5x^2-6x)^{-1/2} \cdot$

$\cdot (10x-6)$

b) $f(x) = 7^{x^3+6x-5} \cdot \ln\left(tg\,(8x^2+2x)\right)$

$f'(x) = 7^{x^3+6x-5} \cdot (3x^2+6) \cdot \ln 7 \cdot \ln\left(tg\,(8x^2+2x)\right) + 7^{x^3+6x-5} \cdot \dfrac{1}{tg\,(8x^2+2x)} \cdot$

$\cdot \left(1+tg^2(8x^2+2x)\right) \cdot (16x+2)$

c) $f(x) = \dfrac{\text{arc}\cos\left(e^{7x^2+6x-1}\right)\text{sen}\,(6x^2-x)}{\left(\log_5(3x-5)\right)}$

$f'(x) = \dfrac{\left[\dfrac{-1}{\sqrt{1-\left(e^{7x^2+6x-1}\right)^2}} \cdot e^{7x^2+6x-1} \cdot (14x+6) \cdot \text{sen}\,(6x^2-x) + \text{arc}\cos\left(e^{7x^2+6x-1}\right)\cos(6x^2-x)\right.}{\left[\log_5(3x-5)\right]^2}$

$\dfrac{\left.\cdot (12x-1)\right] \cdot \log_5(3x-5) - \left[\text{arc}\cos\left(e^{7x^2+6x-1}\right) \cdot \text{sen}\,(6x^2-x) \cdot \dfrac{1}{3x-5} \cdot 3 \cdot \dfrac{1}{\ln 5}\right]}{\left[\log_5(3x-5)\right]^2}$

d) $f(x) = \dfrac{\cos(4x^2+5x) \cdot \ln(3^{x^4})}{\text{sen}\left(\log_7(x^2-2)\right)}$

$f'(x) = \dfrac{\left[\left[-\text{sen}\,(4x^2+5x) \cdot (8x+5)\right] \cdot \ln(3^{x^4}) + \cos(4x^2+5x) \cdot \dfrac{1}{3^{x^4}} \cdot 3^{x^4} \cdot 4x^3 \cdot \ln 3\right] \cdot \text{sen}\left(\log_7(x^2-2)\right)}{\left[\text{sen}\left(\log_7(x^2-2)\right)\right]^2}$

$\dfrac{-\cos(4x^2+5x) \cdot \ln(3^{x^4}) \cdot \cos\left(\log_7(x^2-2)\right) \cdot \dfrac{1}{x^2-2} \cdot 2x \cdot \dfrac{1}{\ln 7}}{\left[\text{sen}\left(\log_7(x^2-2)\right)\right]^2}$

e) $f(x) = \ln^6 (\text{arc tg} \sqrt[5]{2x+5})$

$$f'(x) = 6 \{\ln^5 (\text{arc tg} \sqrt[5]{2x+5})\} \frac{1}{\text{arc tg} \sqrt[5]{2x+5}} \cdot \frac{1}{1+(\sqrt[5]{2x+5})^2} \cdot \frac{1}{5} (2x+5)^{-4/5} \cdot 2$$

(3) $f(x) = \begin{cases} x^2 + ax - 5 & \text{si } x < 2 \\ \frac{1}{x-1} + b & \text{si } x \geq 2 \end{cases}$

Cont en su int. def. por ser polinómica
" " " " " porque $1 \notin [2, +\infty)$

En $x=2$

$f(2) = \frac{1}{2-1} + b = 1+b$

$\lim_{x \to 2^-} x^2 + ax - 5 = 4 + 2a - 5 = -1 + 2a$

$\lim_{x \to 2^+} \frac{1}{x-1} + b = 1 + b$

$-1 + 2a = 1 + b$

$\boxed{2a - b = 2}$

$f'(x) = \begin{cases} 2x + a & \text{si } x < 2 \\ -\frac{1}{(x-1)^2} & \text{si } x \geq 2 \end{cases}$

$f'(2^-) = 4 + a$

$f'(2^+) = -\frac{1}{1} = -1$

$\boxed{4 + a = -1}$

$\left.\begin{matrix} 2a - b = 2 \\ 4 + a = -1 \end{matrix}\right\}$ $\boxed{a = -5}$

$-10 - b = 2 \rightarrow \boxed{b = -12}$

Si $a = -5$, $b = -12$, $f(x)$ continua en $\mathbb{R}$

(4) $f(x) = \begin{cases} \frac{1}{x+3} & \text{si } x \leq 0 \\ -x^2 + 14 & \text{si } 0 < x \leq 3 \\ \sqrt{x^2+16} & \text{si } x > 3 \end{cases}$

Disc. evitable en $x = -3$
Cont. por ser polinómica
Cont pq $x^2 + 16 \geq 0$ siempre

$f'(x) = \begin{cases} -\frac{1}{(x+3)^2} & \text{si } x \leq 0 \\ -2x & \text{si } 0 < x \leq 3 \\ \frac{2x}{2\sqrt{x^2+16}} & \text{si } x > 3 \end{cases}$

Der.

En $x=0$

$f(0) = \frac{1}{3}$

$\lim_{x \to 0^-} \frac{1}{x+3} = \frac{1}{3}$

$\lim_{x \to 0^+} -x^2 + 14 = 14$

Disc inevitable de salto finito en $x=0$

$f'(0^-) = -\frac{1}{9}$

$f'(0^+) = 0$

No es derivable en $x = 0$

En $x = 3$

$f(3) = -6$

$\lim_{x \to 3^-} -x^2 + 14 = 5$

$\lim_{x \to 3^+} \sqrt{x^2+16} = 5$

Cont. en $x = 3$

$f'(3^-) = -6$

$f'(3^+) = \frac{6}{2\sqrt{9+16}} = \frac{6}{10}$

No es derivable en $x = 3$