69 a) $\int x^3 \ln x \, dx = \frac{x^4}{4} \cdot \ln x - \int \frac{x^4}{4} \cdot \frac{1}{x} dx = \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 dx =$

$$\boxed{\begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = x^3 dx \rightarrow v = \frac{x^4}{4} \end{array}}$$

$= \frac{x^4}{4} \ln x - \frac{1}{4} \frac{x^4}{4} + k = \frac{x^4}{4} \ln x - \frac{x^4}{16} + k$

b) $\int \ln(2x+1) \, dx = x \cdot \ln|2x+1| - \int x \cdot \frac{2}{2x+1} dx = x \ln|2x+1| - \int \frac{2x}{2x+1} dx =$

$$\boxed{\begin{array}{l} u = \ln(2x+1) \rightarrow du = \frac{2}{2x+1} dx \\ dv = dx \rightarrow v = x \end{array}}$$

$$\begin{array}{r|l} 2x & 2x+1 \\ \hline -2x-1 & 1 \\ -1 & \end{array}$$

$= x \ln|2x+1| - \int \left(1 - \frac{1}{2x+1}\right) dx = x \ln|2x+1| - \left[x - \frac{1}{2} \ln|2x+1|\right] =$

$= x \ln|2x+1| - x + \frac{1}{2} \ln|2x+1| + k$

c) $\underbrace{\int e^{-x} \operatorname{sen} 2x \, dx}_{I} = -\frac{e^{-x} \cos 2x}{2} - \int -\frac{1}{2} \cos 2x \cdot (-e^{-x}) dx =$

$$\boxed{\begin{array}{l} u = e^{-x} \rightarrow du = -e^{-x} dx \\ dv = \operatorname{sen} 2x \, dx \rightarrow v = -\frac{1}{2} \cos 2x \end{array}}$$

$= -\frac{e^{-x} \cos 2x}{2} - \frac{1}{2}\left[\int e^{-x} \cos 2x \, dx\right] = -\frac{e^{-x} \cos 2x}{2} - \frac{1}{2}\left[\frac{e^{-x} \operatorname{sen} 2x}{2} - \int \frac{1}{2} \operatorname{sen} 2x \, e^{-x} dx\right]$

$u = e^{-x} \rightarrow du = -e^{-x} dx$

$dv = \cos 2x \, dx \rightarrow v = \frac{1}{2} \operatorname{sen} 2x$

$= \frac{-e^{-x} \cos 2x}{2} - \frac{e^{-x} \operatorname{sen} 2x}{4} + \frac{1}{4} \underbrace{\int \operatorname{sen} 2x \, e^{-x} dx}_{I}$

$I = -\frac{e^{-x} \cos 2x}{2} - \frac{e^{-x} \operatorname{sen} 2x}{4} + \frac{1}{4} I$

$\frac{3}{4} I = -\frac{e^{-x} \cos 2x}{2} - \frac{e^{-x} \operatorname{sen} 2x}{4}$

$I = \frac{4}{3}\left[-\frac{e^{-x} \cos 2x}{2} - \frac{e^{-x} \operatorname{sen} 2x}{4}\right] + k.$

81 a) $\displaystyle\int \frac{3}{x^2-2x+1}\,dx = \int \frac{3}{(x-1)^2}\,dx = \int 3\,(x-1)^{-2}\,dx = 3\int (x-1)^{-2}\,dx =$

$\displaystyle = 3\,\frac{(x-1)^{-1}}{-1} + k = -\frac{3}{(x-1)} + k$

b) $\displaystyle\int \frac{x+2}{x^2-2x+1}\,dx$

$\displaystyle\frac{x+2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{A(x-1)+B}{(x-1)^2}$

$\begin{array}{lll} \text{Si} \quad x=1 & 3 = B & \\ \text{Si} \quad x=0 & 2 = -A+B & \end{array} \Bigg\} \begin{array}{l} B=3 \\ A=1 \end{array}$

$\displaystyle\int \frac{x+2}{x^2-2x+1}\,dx = \int \left(\frac{1}{x-1} + \frac{3}{(x-1)^2}\right) dx = \int \frac{1}{x-1}\,dx + 3\int \frac{1}{(x-1)^2}\,dx =$

$\displaystyle = \ln|x-1| - \frac{3}{x-1} + k$

c) $\displaystyle\int \frac{x}{x^2-2x+1}\,dx$

$\displaystyle\frac{x}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{A(x-1)+B}{(x-1)^2}$

$\begin{array}{lll} \text{Si} \quad x=1 & 1 = B & \\ \quad\quad x=0 & 0 = -A+B & \end{array} \Bigg\} \begin{array}{l} A=1 \\ B=1 \end{array}$

$\displaystyle\int \frac{x}{x^2-2x+1}\,dx = \int \frac{1}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx = \ln|x-1| + \int (x-1)^{-2}\,dx =$

$\displaystyle = \ln|x-1| + \frac{(x-1)^{-1}}{-1} + k = \ln|x-1| - \frac{1}{x-1} + k$

d) $\displaystyle\int \frac{x^2}{x^2-2x+1}\,dx = \int \left(1 + \frac{2x-1}{x^2-2x+1}\right) dx =$ (*)

$\begin{array}{r|l} \cancel{x^2} & x^2-2x+1 \\ \hline -\cancel{x^2}+2x-1 & 1 \\ \hline 2x-1 & \end{array}$

$\displaystyle\frac{2x-1}{(x-1)^2}\,dx = \frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{A(x-1)+B}{(x-1)^2}$

$\begin{array}{lll} \text{Si} \quad x=1 & 1 = B & \\ \quad\quad x=0 & -1 = -A+B & \end{array} \Bigg\} \begin{array}{l} B=1 \\ A=2 \end{array}$

(*) $\displaystyle = \int 1\,dx + \int \frac{2}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx = x + 2\ln|x-1| - \frac{1}{x-1} + k.$

d) $\int \operatorname{arc\,tg} x \, dx = x \operatorname{arc\,tg} x - \int x \cdot \frac{1}{1+x^2} dx = x \operatorname{arc\,tg} x - \frac{1}{2}\int \frac{2x}{1+x^2} dx =$

$u = \operatorname{arc\,tg} x \rightarrow du = \frac{1}{1+x^2} dx$

$dv = dx \rightarrow v = x$

$= x \operatorname{arc\,tg} x - \frac{1}{2} \ln|1+x^2| + k$

e) $\int \overbrace{\frac{\ln x}{x}}^{I} dx = [\ln|x| \cdot \ln|x|] - \int \overbrace{\frac{\ln x}{x}}^{I} \cdot dx \rightarrow$

$u = \ln x \rightarrow du = \frac{1}{x} dx$

$dv = \int \frac{1}{x} dx \rightarrow v = \ln x$

$2I = [\ln|x|]^2 \rightarrow I = \frac{\ln^2|x|}{2} + k$

También se puede hacer un cambio de variable

$\int \frac{\ln x}{x} dx = \int u \, du = \frac{u^2}{2} + k = \frac{\ln^2 x}{2} + k$

$u = \ln x \rightarrow du = \frac{1}{x} dx$

f) $\int x \operatorname{sen} 2x \, dx = \left(-\frac{1}{2}\cos 2x\right) \cdot x - \int -\frac{1}{2}\cos 2x \, dx =$

$u = x \rightarrow du = dx$

$dv = \operatorname{sen} 2x \, dx \rightarrow v = -\frac{1}{2}\cos 2x$

$= -\frac{x \cdot \cos 2x}{2} + \frac{1}{2} \cdot \frac{1}{2}\int 2\cos 2x \, dx = \frac{-x\cos 2x}{2} + \frac{1}{4}\operatorname{sen} 2x + k$

g) $\int x^2 \operatorname{sen} 2x \, dx = -\frac{x^2 \cos 2x}{2} - \int \left(-\frac{1}{2}\cos 2x\right) 2x \, dx =$

$u = x^2 \rightarrow du = 2x \, dx$

$dv = \operatorname{sen} 2x \, dx \rightarrow v = -\frac{1}{2}\cos 2x$

$= \frac{-x^2 \cos 2x}{2} + \int x \cos 2x \, dx =$

$u = x \rightarrow du = dx$

$dv = \cos 2x \, dx \rightarrow v = \frac{1}{2}\operatorname{sen} 2x$

$= \frac{-x^2\cos 2x}{2} + \frac{x \operatorname{sen} 2x}{2} - \int \frac{1}{2}\operatorname{sen} 2x \, dx =$

$= \frac{-x^2\cos 2x}{2} + \frac{x \operatorname{sen} 2x}{2} + \frac{1}{4}\cos 2x + k.$

h) $\int (2x+3)\, e^{2x}\, dx = \dfrac{e^{2x}.(2x+3)}{2} - \int \frac{1}{2} e^{2x}.2dx =$

$\boxed{\begin{array}{l} u=(2x+3) \rightarrow du = 2\,dx \\ dv = e^{2x}\,dx \rightarrow v = \frac{1}{2}e^{2x} \end{array}}$

$= \dfrac{e^{2x}(2x+3)}{2} - \frac{1}{2} e^{2x} + k.$

i) $\int \dfrac{x}{e^x}\, dx = \int x.e^{-x}\, dx = -xe^{-x} - \int -e^{-x}\, dx = -xe^{-x} - e^{-x} + k$

$\boxed{\begin{array}{l} u=x \rightarrow du = dx \\ dv = e^{-x}dx \rightarrow v = -e^{-x} \end{array}}$

j) $\int (x^2-5)\cos x\, dx = (x^2-5)\,\mathrm{sen}\,x - \int \mathrm{sen}\,x \,.\, 2x\, dx =$

$\boxed{\begin{array}{l} u=x^2-5 \rightarrow du = 2x\,dx \\ dv = \cos x\,dx \rightarrow v = \mathrm{sen}\,x \end{array}}$ $\boxed{\begin{array}{l} u=2x \rightarrow du = 2\,dx \\ dv = \mathrm{sen}\,x\,dx \rightarrow v = -\cos x \end{array}}$

$= (x^2-5)\,\mathrm{sen}\,x - \left[2x(-\cos x) - \int -\cos x \,.\, 2dx\right] =$

$= (x^2-5)\,\mathrm{sen}\,x + 2x\cos x - 2\,\mathrm{sen}\,x + k$

k) $\int (2x^2+x-2)\, e^{3x}\, dx = \dfrac{(2x^2+x-2)e^{3x}}{3} - \int \frac{1}{3} e^{3x}.(4x+1)\, dx =$

$\boxed{\begin{array}{l} u=2x^2+x-2 \rightarrow du = (4x+1)dx \\ dv = e^{3x}dx \rightarrow v = \frac{1}{3}e^{3x} \end{array}}$ $\boxed{\begin{array}{l} u=4x+1 \rightarrow du = 4\,dx \\ dv = \frac{1}{3}e^{3x}dx \rightarrow v = \frac{1}{9}e^{3x} \end{array}}$

$= \dfrac{(2x^2+x-2)e^{3x}}{3} - \dfrac{(4x+1)e^{3x}}{9} + \int \frac{1}{9} e^{3x}.4\,dx =$

$= \dfrac{(2x^2+x-2)e^{3x}}{3} - \dfrac{(4x+1)e^{3x}}{9} + \frac{4}{9}\cdot\frac{1}{3} e^{3x} + k$

l) $\int (2+e^{2x})\cos(x+1)\, dx = \int 2\cos(x+1)\,dx + \int e^{2x}\cos(x+1)\, dx =$

$= 2\,\mathrm{sen}\,(x+1) + \int e^{2x}\cos(x+1)\,dx = ⊛$

$\underbrace{\int e^{2x}\cos(x+1)\,dx}_{I} = e^{2x}.\mathrm{sen}\,(x+1) - \int \mathrm{sen}\,(x+1)\, 2e^{2x}\,dx =$

$\boxed{\begin{array}{l} u=e^{2x} \rightarrow du = 2e^{2x}dx \\ dv = \cos(x+1)\,dx \rightarrow v = \mathrm{sen}\,(x+1) \end{array}}$ $\boxed{\begin{array}{l} u=e^{2x} \rightarrow du = 2e^{2x}dx \\ dv = \mathrm{sen}\,(x+1)\,dx \rightarrow v = -\cos(x+1) \end{array}}$

$= e^{2x}\,\mathrm{sen}\,(x+1) - 2\left[e^{2x}.(-\cos(x+1)) - \int -\cos(x+1).2e^{2x}dx\right] =$

$= e^{2x}\,\mathrm{sen}\,(x+1) + 2e^{2x}\cos(x+1) - 4\underbrace{\int e^{2x}\cos(x+1)\,dx}_{I}$

$5I = e^{2x}\,\mathrm{sen}\,(x+1) + 2e^{2x}\cos(x+1) \longrightarrow I = \dfrac{e^{2x}\,\mathrm{sen}(x+1) + 2e^{2x}\cos(x+1)}{5}$

⊛ $= 2\,\mathrm{sen}\,(x+1) + \dfrac{e^{2x}\,\mathrm{sen}\,(x+1) + 2e^{2x}\cos(x+1)}{5} + k.$