CONTROL LOGARITMOS Y EXPONENCIALES

1. Resuelve: $9^x - 2 \cdot 3^{x+2} + 81 = 0$

2. Resuelve: $2\log(3x-2) - 1 = \log(x+6)$

3. Resuelve: $\begin{cases} x + y = 22 \\ \log x - \log y = 1 \end{cases}$

4. Si log 2=0,3010, log 3= 0,4771, log 5= 0,6990. Calcula:

   a. $\log \sqrt[5]{\frac{1}{0,64}}$ b. $\log \frac{270}{128}$

5. Escribe la expresión algebraica de: $\log A = 3 + 3\log x - \frac{2}{5}\log(y) - 5\log z - 4\log\frac{x}{y}$

6. Resuelve: $\left.\begin{array}{r} \log x + \log y = 3 \\ x + \frac{y}{10} = 20 \end{array}\right\}$

7. Resuelve $\frac{\log 2 + \log(11 - x^2)}{\log(5-x)} = 2$

8. Resuelve $8^{1+x} + 2^{3x-1} = \frac{17}{16}$

CONTROL LOGARITMOS Y EXPONENCIALES 1ºBach B

(1) $9^x - 2\cdot 3^{x+2} + 81 = 0 \rightarrow (3^x)^2 - 2\cdot 3^x \cdot 3^2 + 81 = 0 \rightarrow$

$(3^x)^2 - 18\cdot 3^x + 81 = 0 \xrightarrow{3^x = t} t^2 - 18t + 81 = 0$

$t = \frac{18 \pm \sqrt{18^2 - 4\cdot 81}}{2} = 9 \rightarrow 3^x = 9 \rightarrow \boxed{x=2}$

(2) $2\log(3x-2) - 1 = \log(x+6)$

$\log(3x-2)^2 - \log 10 = \log(x+6) \rightarrow \cancel{\log}\frac{(3x-2)^2}{10} = \cancel{\log}(x+6) \Rightarrow$

$(3x-2)^2 = 10(x+6) \rightarrow 9x^2 - 12x + 4 = 10x + 60 \rightarrow$

$9x^2 - 22x - 56 = 0 \rightarrow x = 4$

$\rightarrow x = -\frac{14}{9}$ No vale.

(3) $\left.\begin{array}{l} x+y=22 \\ \log x - \log y = 1 \end{array}\right\} \left.\begin{array}{l} x+y=22 \\ \log\frac{x}{y} = \log 10 \end{array}\right\} \left.\begin{array}{l} x+y=22 \\ x = 10y \end{array}\right\} \begin{array}{l} 11y = 22 \\ y = 2 \\ x = 20 \end{array}$

(4) $\log 2 = 0{,}3010$; $\log 3 = 0{,}4771$; $\log 5 = 0{,}6990$

a) $\log\sqrt[5]{\frac{1}{0{,}64}} = \frac{1}{5}\log\frac{100}{64} = \frac{1}{5}\left[\log 100 - \log 64\right] =$

$= \frac{1}{5}\left[2 - \log 2^6\right] = \frac{1}{5}\left[2 - 6\cdot\log 2\right] = \frac{1}{5}\left[2 - 6\cdot 0{,}3010\right] =$

$= \frac{97}{2500} = 0{,}0388$

b) $\log\frac{270}{128} = \log 270 - \log 128 = \log 3^3\cdot 2\cdot 5 - \log 2^7 =$

$= 3\log 3 + \log 2 + \log 5 - 7\log 2 = 0{,}3243$

(5) $\log A = \log 1000 + \log x^3 - \log y^{2/5} - \log z^5 - \log\left(\frac{x}{y}\right)^4$

$\cancel{\log} A = \cancel{\log}\frac{1000\cdot x^3}{\sqrt[5]{y^2}\cdot z^5\cdot\frac{x^4}{y^4}} \rightarrow A = \frac{1000\, x^3 y^4}{\sqrt[5]{y^2}\, z^5\, x^4} = \frac{1000\cdot y^{18/5}}{z^5\cdot x}$

6) $\log x + \log y = 3$; $x + \frac{y}{10} = 20$ $\Big\{$ $\cancel{\log} xy = \cancel{\log} 1000$

$xy = 1000$

$10x + y = 200$ $\Big\{$ $y = 200 - 10x$

$x(200 - 10x) = 1000$

$200x - 10x^2 = 1000$

$10x^2 - 200x + 1000 = 0 \rightarrow x^2 - 20x + 100 = 0 \longrightarrow x = 10$

$y = 200 - 100 = 100$

7) $\dfrac{\log 2 + \log(11 - x^2)}{\log(5-x)} = 2 \Rightarrow \log[2(11 - x^2)] = 2\log(5-x) \Rightarrow$

$\cancel{\log}[22 - 2x^2] = \cancel{\log}(5-x)^2 \rightarrow 22 - 2x^2 = 25 - 10x + x^2$

$3x^2 - 10x + 3 = 0 \rightarrow x = 3$

$\rightarrow x = \frac{1}{3}$

8) $8^{1+x} + 2^{3x-1} = \frac{17}{16}$

$8 \cdot 8^x + (2^x)^3 \cdot \frac{1}{2} = \frac{17}{16} \rightarrow \dfrac{8(2^x)^3 \cdot 2 + (2^x)^3}{2} = \frac{17}{16}$

$2^x = t \longrightarrow 16t^3 + t^3 = \frac{34}{16} \rightarrow 17t^3 = \frac{34}{16} \rightarrow t^3 = \frac{2}{16} = \frac{1}{8}$

$t^3 = 2^{-3} = \left(\frac{1}{2}\right)^3 \rightarrow t = \frac{1}{2}$

$2^x = \frac{1}{2} = 2^{-1} \rightarrow \boxed{x = -1}$

Escaneado con CamScanne

CONTROL LOGARITMOS Y EXPONENCIALES

1. Resuelve: $3^{2(x+1)} - 28 \cdot 3^x + 3 = 0$

2. Resuelve: $2 \log x - 1 = \log (x^2 - 6)$

3. Resuelve: $\begin{cases} x - y = 3 \\ 2^x - 2^y = \frac{7}{4} \end{cases}$

4. Si log 2=0,3010, log 3= 0,4771, log 5= 0,6990. Calcula:

a. $\log \sqrt[6]{\frac{1}{0{,}128}}$ b. $\log \frac{540}{256}$

5. Escribe la expresión algebraica de: $\log A = 2 + 5 \log x - \frac{3}{5} \log(y) - 4 \log z - 3 \log \frac{x}{y}$

6.Resuelve: $\left. \begin{array}{c} \log x - \log y = 1 \\ 3x + 2y = 64 \end{array} \right\}$

7. Resuelve $\frac{\log 2 + \log(11 - x^2)}{\log(5-x)} = 2$

8. Resuelve $4^{1+x} + 2^{x+3} = 320$

CONTROL LOGARITMOS Y EXPONENCIALES (2)

① $3^{2(x+1)} - 28 \cdot 3^x + 3 = 0$

$3^{2x+2} - 28 \cdot 3^x + 3 = 0 \rightarrow (3^x)^2 \cdot 3^2 - 28 \cdot 3^x + 3 = 0 \xrightarrow{3^x = t}$

$9t^2 - 28t + 3 = 0 \rightarrow t = \dfrac{28 \pm \sqrt{28^2 - 4 \cdot 9 \cdot 3}}{18} = \begin{cases} 3 \\ 1/9 \end{cases}$

$t_1 = 3 \rightarrow 3^x = 3 \rightarrow x = 1$

$t_2 = \frac{1}{9} \rightarrow 3^x = 3^{-2} \rightarrow x = -2$

② $2 \log x - 1 = \log(x^2 - 6)$

$\log x^2 - \log 10 = \log(x^2 - 6) \rightarrow \cancel{\log} \dfrac{x^2}{10} = \cancel{\log}(x^2 - 6) \rightarrow \dfrac{x^2}{10} = x^2 - 6$

$x^2 = 10x^2 - 60 \rightarrow 60 = 9x^2 \rightarrow x = \pm\sqrt{\dfrac{60}{9}}$ solo vale $x = +\sqrt{\dfrac{60}{9}} = \dfrac{2\sqrt{15}}{3}$

③ $\begin{cases} x - y = 3 \\ 2^x - 2^y = \frac{7}{4} \end{cases}$ $\begin{cases} x = 3 + y \\ 2^{3+y} - 2^y = \frac{7}{4} \end{cases} \rightarrow 2^3 \cdot 2^y - 2^y = \frac{7}{4} \xrightarrow{2^y = t}$

$8t - t = \frac{7}{4} \rightarrow 7t = \frac{7}{4} \rightarrow t = \frac{1}{4} \rightarrow 2^y = \frac{1}{4} = 2^{-2} \rightarrow y = -2$

$x = 3 - 2 = 1 \rightarrow \boxed{\begin{matrix} x = 1 \\ y = -2 \end{matrix}}$

④ $\log 2 = 0{,}3010$, $\log 3 = 0{,}4771$, $\log 5 = 0{,}6990$

a) $\log \sqrt[6]{\dfrac{1}{0{,}128}} = \log \sqrt[6]{\dfrac{125}{16}} = \dfrac{1}{6} \log \dfrac{125}{16} = \dfrac{1}{6}\left[\log 125 - \log 16\right] =$

$= \dfrac{1}{6}\left[\log 5^3 - \log 2^4\right] = \dfrac{1}{6}\left[3 \log 5 - 4 \log 2\right] =$

$= \dfrac{1}{6}\left[3 \cdot 0{,}6990 - 4 \cdot 0{,}3010\right] = 0{,}1488$

b) $\log \dfrac{540}{256} = \log \dfrac{2^2\, 3^3\, 5}{2^8} = \log 2^2 \cdot 3^3 \cdot 5 - \log 2^8 =$

$= 2 \log 2 + 3 \log 3 + \log 5 - 8 \cdot \log 2 = 0{,}3243$

⑤ $\log A = 2 + 5 \log x + \frac{3}{5} \log y - 4 \log z - 3 \log \frac{x}{y}$

$\log A = \log 100 + \log x^5 + \log \sqrt[5]{y^3} - \log z^4 - \log\left(\frac{x}{y}\right)^3$

$A = \dfrac{100 \cdot x^5 \cdot \sqrt[5]{y^3}}{z^4 \cdot \dfrac{x^3}{y^3}} = \dfrac{100\, x^2 \cdot y^3 \sqrt[5]{y^3}}{z^4}$

6. $\left.\begin{array}{l}\log x - \log y = 1 \\ 3x + 2y = 64\end{array}\right\}$ $\left.\begin{array}{l}\log \frac{x}{y} = \log 10 \\ 3x + 2y = 64\end{array}\right\}$ $\begin{array}{l}x = 10y \\ 3 \cdot 10y + 2y = 64\end{array}$

$32y = 64 \rightarrow \boxed{y = 2}$

$\boxed{x = 20}$

7. $\dfrac{\log 2 + \log(11 - x^2)}{\log(5 - x)} = 2$

$\log 2 + \log(11 - x^2) = 2\log(5 - x)$

$\cancel{\log} 2(11 - x^2) = \cancel{\log}(5 - x)^2$

$22 - 2x^2 = 25 - 10x + x^2$

$-3x^2 + 10x - 3 = 0$ $\begin{array}{l}\nearrow x_1 = 3 \\ \searrow x_2 = \frac{1}{3}\end{array}$

8. $4^{1+x} + 2^{x+3} = 320$

$(2^2)^{1+x} + 2^{x+3} = 320 \longrightarrow 2^2 \cdot (2^x)^2 + 2^x \cdot 2^3 = 320 \rightarrow \boxed{2^x = t}$

$4t^2 + 8t - 320 = 0$ $\begin{array}{l}\nearrow t_1 = 8 \longrightarrow 2^x = 8 \rightarrow x = 3 \\ \searrow t_2 = -10 \rightarrow 2^x = -10 \ \not\exists\end{array}$