

TEMA 7. 1º BACHILLERATO A

1. a) Pasar a forma polar y trigonométrica: $-3 - \sqrt{7}i$

b) Pasar a forma binómica y trigonométrica: $\sqrt{5}_{145^\circ}$

c) Pasar a forma binómica y polar $z = 3(\cos 25^\circ + i \sin 25^\circ)$ (1,5 puntos)

2. Calcular m y n para que se cumpla:

$$\frac{m-2i}{3+ni} = \sqrt{2}_{75^\circ} \text{ (1,5 puntos)}$$

3. Resuelve la ecuación $z^8 = -2 + 9i$ (1,5 puntos)

4. Calcula:

$$b) \left(\frac{(3+2i)}{5-7i} \right)^{30} =$$

$$b) \frac{(5_{95^\circ} \cdot 7_{65^\circ})^5}{3_{135^\circ}} = (1,5 \text{ puntos})$$

5. Calcula las raíces quintas de $-13 + 3\sqrt{5}i$ (1,5 puntos)

6. Calcula:

$$b) \frac{i^7 - i^6 + i^{26} - i^{53}}{3 - i^9} =$$

$$b) \frac{5i^{17} - 7i^5 + (3i)^6}{4i^5 + i^{19}} = (1,5 \text{ puntos})$$

7. Resuelve: $(2+3i) \cdot (5-i) + 8z = z(7-2i)$ (1 punto)

TEMA 7 1' Doch A

(1,5) ①

a) $z = -3 - \sqrt{7}i$

$r = \sqrt{(-3)^2 + (\sqrt{7})^2} = \sqrt{16} = 4$ $\text{tg } \alpha = \frac{-\sqrt{7}}{-3} \Rightarrow \alpha = 221,41^\circ$

$z = 4_{221,41^\circ} = 4 (\cos 221,41^\circ + i \text{sen } 221,41^\circ)$

b) $z = \sqrt{5}_{145^\circ} =$

$a = r \cos \alpha = \sqrt{5} \cdot \cos 145^\circ = -1,83$

$b = r \text{sen } \alpha = \sqrt{5} \cdot \text{sen } 145^\circ = 1,28$

$z = -1,83 + 1,28i = \sqrt{5} (\cos 145^\circ + i \text{sen } 145^\circ)$

c) $z = 3 (\cos 25^\circ + i \text{sen } 25^\circ)$

$a = 3 \cos 25^\circ = 2,72$

$b = 3 \text{sen } 25^\circ = 1,27$

$z = 3_{25^\circ} = 2,72 + 1,27i$

(1,5) ②

$\frac{m-2i}{3+ni} = \sqrt{2}_{75^\circ}$

$\sqrt{2}_{75^\circ} = 0,37 + 1,37i$

$(m-2i) = (3+ni) (0,37 + 1,37i) = 1,11 + 4,11i + 0,37ni - 1,37n$

$m-2i = (1,11 - 1,37n) + (4,11 + 0,37n)i$

$m = 1,11 - 1,37n$ $m = 23,73$

$-2 = 4,11 + 0,37n$ $n = -16,51$

(1,5) ③

$z^8 = -2 + 9i$

$r = \sqrt{(-2)^2 + 9^2} = \sqrt{85}$

$\text{tg } \alpha = \frac{9}{-2} \Rightarrow \alpha = 102,53^\circ$

$360^\circ : 8 = 45^\circ$

$S = \sqrt[8]{\sqrt{85}} = \sqrt[16]{85}$

$102,53^\circ : 8 = 12,82^\circ$

$\sqrt[16]{85}_{12,82^\circ}; \sqrt[16]{85}_{57,82^\circ}; \sqrt[16]{85}_{102,82^\circ}; \sqrt[16]{85}_{147,82^\circ}; \sqrt[16]{85}_{192,82^\circ};$

$\sqrt[16]{85}_{237,82^\circ}; \sqrt[16]{85}_{282,82^\circ}; \sqrt[16]{85}_{327,82^\circ}$

(1,5) ④

a) $\left(\frac{3+2i}{5-7i}\right)^{30} = \left(\frac{(3+2i)(5+7i)}{(5-7i)(5+7i)}\right)^{30} = \left(\frac{15+21i+10i+14i^2}{25-49i^2}\right)^{30} = \left(\frac{1+32i}{74}\right)^{30} =$

$= \left(\frac{1}{74} + \frac{31}{74}i\right)^{30} = \left(\frac{\sqrt{962}}{74} 88,21^\circ\right)^{30} = \left(\frac{\sqrt{962}}{74}\right)^{30} 126,3^\circ$

$r = \sqrt{\left(\frac{1}{74}\right)^2 + \left(\frac{31}{74}\right)^2} = \frac{\sqrt{962}}{74}$ $\text{tg } \alpha = \frac{31/74}{1/74} \rightarrow \alpha = 88,21$

* Otra forma * $3+2i = \sqrt{13}_{33,69^\circ}$ $5-7i = \sqrt{74}_{305,53^\circ}$

$\left(\frac{3+2i}{5-7i}\right)^{30} = \left(\frac{\sqrt{13}_{33,69^\circ}}{\sqrt{74}_{305,53^\circ}}\right)^{30} = \left(\frac{\sqrt{13}}{\sqrt{74}} -271,84^\circ\right)^{30} = \left(\frac{\sqrt{13}}{74} 88,16^\circ\right)^{30} = \left(\frac{\sqrt{13}}{74}\right)^{30} 126,30^\circ$

$$b) \frac{(595 \cdot 765)^5}{3^{135}} = \frac{(35160)^5}{3^{135}} = \frac{52521875}{3} \frac{800}{135} = 17507291,67 \cdot 305^\circ$$

$$1.5 \text{ (5)} \quad z = \sqrt[5]{-13 + 3\sqrt{5}i}$$

$$r = \sqrt{(-13)^2 + (3\sqrt{5})^2} = \sqrt{214} \quad \text{tg } \alpha = \frac{3\sqrt{5}}{-13} \rightarrow \alpha = 152,71^\circ$$

$$152,71^\circ : 5 = 30,54^\circ$$

$$360^\circ : 5 = 72^\circ$$

$$s = \sqrt[5]{\sqrt{214}} = \sqrt[10]{214}$$

$$\sqrt[10]{214}_{30,54^\circ}; \sqrt[10]{214}_{102,54^\circ}; \sqrt[10]{214}_{174,54^\circ}; \sqrt[10]{214}_{246,54^\circ}; \sqrt[10]{214}_{318,54^\circ}$$

$$(1.5) \text{ (6)} \quad a) \frac{i^7 - i^6 + i^{26} - i^{53}}{3 - i^9} = \frac{i^3 - i^2 + i^2 - i}{3 - i} = \frac{-i - (-1) + (-1) - i}{3 - i} = \frac{-2i}{3 - i} =$$

$$= \frac{-2i(3+i)}{(3-i)(3+i)} = \frac{2-6i}{10} = \frac{1}{5} - \frac{3}{5}i$$

$$b) \frac{5i^{17} - 7i^5 + (3i)^6}{4i^5 + i^{19}} = \frac{5i - 7i + 729i^2}{4i + i^3} = \frac{-729 - 2i}{3i} = \frac{(-729 - 2i)(-3i)}{(3i)(-3i)} =$$

$$= \frac{2187i + 6i^2}{-9i^2} = \frac{-6 + 2187i}{+9} = -\frac{2}{3} + 243i$$

$$(1) \text{ (7)} \quad (2+3i)(5-i) + 8z = z(7-2i)$$

$$10 - 2i + 15i - 3i^2 + 8z = z(7-2i)$$

$$13 + 13i + 8z = z(7-2i)$$

$$8z - z(7-2i) = -13 - 13i$$

$$z(8-7+2i) = -13 - 13i$$

$$z(1+2i) = -13 - 13i$$

$$z = \frac{-13-13i}{1+2i} = \frac{(-13-13i)(1-2i)}{(1+2i)(1-2i)} = \frac{-13+26i-13i+26i^2}{1-4i^2} =$$

$$= \frac{-39+13i}{5} = -\frac{39}{5} + \frac{13}{5}i$$