T.4 2º Bach A

(1) $A(-1,4,-1)$ $\quad \vec{AB}(-2,1,3)$

$B(-3,5,2)$ $\quad \vec{AC}(-1,-3,1)$

$C(-2,1,0)$ $\quad \vec{AD}(-6,-1,0)$

$D(-7,3,-1)$

$$V = \frac{\begin{vmatrix} -2 & 1 & 3 \\ -1 & -3 & 1 \\ -6 & -1 & 0 \end{vmatrix}}{6} = \frac{(3-6)-(54+2)}{6} = \left|\frac{-3-56}{6}\right| = \left|-\frac{59}{6}\right|$$

$$V = \frac{59}{6} u^3 = 9{,}83\, u^3$$

Área

$$A_1 = \frac{|\vec{AC} \times \vec{AB}|}{2} = \frac{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & 1 \\ -2 & 1 & 3 \end{vmatrix}}{2} = \frac{|(-10,1,-7)|}{2} = \frac{\sqrt{100+1+49}}{2} = \frac{\sqrt{150}}{2} u^2 = 6{,}12$$

$$A_2 = \frac{|\vec{AD} \times \vec{AC}|}{2} = \frac{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -6 & -1 & 0 \\ -1 & -3 & 1 \end{vmatrix}}{2} = \frac{|(-1,6,17)|}{2} = \frac{\sqrt{1+36+289}}{2} = \frac{\sqrt{326}}{2} u^2 = 9{,}03$$

$$A_3 = \frac{|\vec{BD} \times \vec{BC}|}{2} = \frac{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & -2 & -3 \\ 1 & -4 & -2 \end{vmatrix}}{2} = \frac{|(-8,11,18)|}{2} = \frac{\sqrt{64+121+324}}{2} = \frac{\sqrt{509}}{2} u^2 = 11{,}28$$

$$A_4 = \frac{|\vec{AD} \times \vec{AB}|}{2} = \frac{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -6 & -1 & 0 \\ -2 & 1 & 3 \end{vmatrix}}{2} = \frac{|(-3,18,-8)|}{2} = \frac{\sqrt{397}}{2} u^2 = 9{,}96$$

$$A_T = 36{,}39\, u^2$$

(2) $\vec{u}(-5,-1,2)$, $\vec{v}(-2,1,-5)$

a) $|\vec{u}| = \sqrt{25+1+4} = \sqrt{30}$

$|\vec{v}| = \sqrt{4+1+25} = \sqrt{30}$

b) $\vec{u}\cdot\vec{v} = (-5,-1,2)(-2,1,-5) = 10-1-10 = -1$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -1 & 2 \\ -2 & 1 & -5 \end{vmatrix} = (3,-29,-7)$$

c) $$\cos\alpha = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \frac{-1}{\sqrt{30}\sqrt{30}} \qquad \alpha = 91{,}91^\circ$$

$$\text{Proy}_{\vec{v}}\vec{u} = \frac{|\vec{u}\cdot\vec{v}|}{|\vec{v}|} = \frac{|-1|}{\sqrt{30}} = \frac{1}{\sqrt{30}} = \frac{\sqrt{30}}{30}$$

d) $(-5,m,-3)(-5,-1,2) = 0 \rightarrow 25 - m - 6 = 0 \rightarrow \boxed{m=19}$

e) $\vec{u}\times\vec{v} = (3,-29,-7) = \vec{w}$

$$\vec{a} = \left(\frac{3}{\sqrt{899}}, \frac{-29}{\sqrt{899}}, \frac{-7}{\sqrt{899}}\right)$$

f) $A = |\vec{u}\times\vec{v}| = \sqrt{899}\, u^2$

(3) $$|\vec{v},\vec{a},\vec{b}| = \begin{vmatrix} x & y & z \\ 2 & -1 & 1 \\ 1 & 0 & 3 \end{vmatrix} = 0 \rightarrow -3x + y + z - 6y = 0 \rightarrow -3x - 5y + z = 0$$

$\vec{v}\cdot\vec{c} = (x,y,z)(2,3,0) = 0 \rightarrow 2x + 3y = 0$

$$\left.\begin{matrix} -3x-5y+z=0 \\ 2x+3y=0 \end{matrix}\right\} \; z=\lambda \quad \left.\begin{matrix} -3x-5y=-\lambda \\ 2x+3y=0 \end{matrix}\right\} \quad \begin{matrix} -6x-10y=-2\lambda \\ 6x+9y=0 \\ \hline -y=-2\lambda \end{matrix} \quad y=2\lambda$$

$$2x+6\lambda=0 \rightarrow x = -\frac{6\lambda}{2} = -3\lambda$$

$\vec{v}(-3\lambda, 2\lambda, \lambda)$ $\forall \lambda\in\mathbb{R}$ $\sim (-3,2,1)$