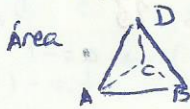


T.4 2. Bach A

① $A(-1, 4, -1)$ $\vec{AB}(-2, 1, 3)$
 $B(-3, 5, 2)$ $\vec{AC}(-1, -3, 1)$
 $C(-2, 1, 0)$ $\vec{AD}(-6, -1, 0)$
 $D(-7, 3, -1)$



$$V = \frac{\begin{vmatrix} -2 & 1 & 3 \\ -1 & -3 & 1 \\ -6 & -1 & 0 \end{vmatrix}}{6} = \frac{(3-6) - (54+2)}{6} = \left| \frac{-3-56}{6} \right| = \left| \frac{-59}{6} \right|$$

$$V = \frac{59}{6} u^3 = 9,83 u^3$$

$$A_1 = \frac{|\vec{AC} \times \vec{AB}|}{2} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 1 \\ -2 & 1 & 3 \end{vmatrix}}{2} = \frac{|(-10, 1, -7)|}{2} = \frac{\sqrt{100+1+49}}{2} = \frac{\sqrt{150}}{2} u^2 = 6,12$$

$$A_2 = \frac{|\vec{AD} \times \vec{AC}|}{2} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & -1 & 0 \\ -1 & -3 & 1 \end{vmatrix}}{2} = \frac{|(-1, 6, 17)|}{2} = \frac{\sqrt{1+36+289}}{2} = \frac{\sqrt{326}}{2} u^2 = 9,03$$

$$A_3 = \frac{|\vec{BD} \times \vec{BC}|}{2} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -2 & -3 \\ 1 & -4 & -2 \end{vmatrix}}{2} = \frac{|(-8, 11, 18)|}{2} = \frac{\sqrt{64+121+324}}{2} = \frac{\sqrt{509}}{2} u^2 = 11,28$$

$$A_4 = \frac{|\vec{AD} \times \vec{AB}|}{2} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & -1 & 0 \\ -2 & 1 & 3 \end{vmatrix}}{2} = \frac{|(-3, 18, -8)|}{2} = \frac{\sqrt{9+324+64}}{2} = \frac{\sqrt{397}}{2} u^2 = 9,96$$

$$A_T = 36,39 u^2$$

② $\vec{u}(-5, -1, 2)$, $\vec{v}(-2, 1, -5)$

a) $|\vec{u}| = \sqrt{25+1+4} = \sqrt{30}$

$|\vec{v}| = \sqrt{4+1+25} = \sqrt{30}$

b) $\vec{u} \cdot \vec{v} = (-5, -1, 2) \cdot (-2, 1, -5) = 10 - 1 - 10 = -1$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -1 & 2 \\ -2 & 1 & -5 \end{vmatrix} = (3, -29, -7)$$

c) $\cos \alpha = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} = \frac{-1}{\sqrt{30} \sqrt{30}} \quad \alpha = 91,91^\circ$

$$\text{Proj}_{\vec{v}} \vec{u} = \frac{(\vec{u} \cdot \vec{v})}{|\vec{v}|} = \frac{-1}{\sqrt{30}} = \frac{1}{\sqrt{30}} = \frac{\sqrt{30}}{30}$$

d) $(-5, m, -3) \cdot (-5, -1, 2) = 0 \rightarrow 25 - m - 6 = 0 \rightarrow \underline{m=19}$

e) $\vec{u} \times \vec{v} = (3, -29, -7) = \vec{w}$
 $\vec{w} = \left(\frac{3}{\sqrt{899}}, \frac{-29}{\sqrt{899}}, \frac{-7}{\sqrt{899}} \right)$

f) $A = |\vec{u} \times \vec{v}| = \sqrt{899} u^2$

③ $|\vec{v}, \vec{a}, \vec{b}| = \begin{vmatrix} x & y & z \\ 2 & -1 & 1 \\ 1 & 0 & 3 \end{vmatrix} = 0 \rightarrow -3x + y + z - 6y = 0 \rightarrow -3x - 5y + z = 0$

$\vec{v} \cdot \vec{c} = (x, y, z) \cdot (2, 3, 0) = 0 \rightarrow 2x + 3y = 0$

$$\begin{cases} -3x - 5y + z = 0 \\ 2x + 3y = 0 \end{cases} \quad \begin{cases} z = \lambda \\ -3x - 5y = -\lambda \end{cases} \quad \begin{cases} -6x - 10y = -2\lambda \\ 6x + 9y = 0 \end{cases}$$

$$-y = -2\lambda \quad y = 2\lambda$$

$$2x + 6\lambda = 0 \rightarrow x = -\frac{6\lambda}{2} = -3\lambda$$

$$\vec{v}(-3\lambda, 2\lambda, \lambda) \quad \forall \lambda \in \mathbb{R} \quad \sim (-3, 2, 1)$$