

TEMA 1. 2º BACHILLERATO B

1. Sean las matrices $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 0 \\ -5 & 2 \end{pmatrix}$. Resuelve la ecuación matricial $AX + A^t \cdot B = C$

2. Sea la matriz $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$. Calcula B^5, B^{12}, B^n

3. Se consideran las matrices $A = \begin{pmatrix} -1 & 0 & 1 \\ 3 & k & 0 \\ -k & 1 & 4 \end{pmatrix}$

a) Calcular los valores de k para los cuales la matriz A no tiene inversa.

b) Estudiar el rango en función de los valores de k .

4. Sea la matriz $A = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$. Calcula las matrices X que verifican $AX = XA$

5. Resuelve el sistema siendo $A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 4 \\ -1 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 2 \\ 3 & -1 & 1 \end{pmatrix}$

$$\left. \begin{array}{l} 2X + 3Y = A \\ 5X - 2Y = 2B \end{array} \right\}$$

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① $AX + A^t \cdot B = C$

$AX = C - A^t \cdot B \rightarrow X = A^{-1} [C - A^t \cdot B]$

$A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \rightarrow \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array} \right) \xrightarrow{2F_1 - F_2} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & -1 & 2 & -1 \end{array} \right) \xrightarrow{F_1 - F_2}$

$\left(\begin{array}{cc|cc} 1 & 0 & -1 & 1 \\ 0 & -1 & 2 & -1 \end{array} \right) \xrightarrow{F_2 / -1} \left(\begin{array}{cc|cc} 1 & 0 & -1 & 1 \\ 0 & 1 & -2 & 1 \end{array} \right)$

$\Rightarrow A^{-1} = \begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix}$

$C - A^t \cdot B = \begin{pmatrix} -1 & 0 \\ -5 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ -5 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 3 \\ 1 & -3 \end{pmatrix} =$

$= \begin{pmatrix} -1 & -3 \\ -6 & 5 \end{pmatrix}$

$X = A^{-1} [C - A^t \cdot B] = \begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} -1 & -3 \\ -6 & 5 \end{pmatrix} = \begin{pmatrix} -5 & 8 \\ -4 & 11 \end{pmatrix}$

② $B = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}, B^5, B^{12}, B^n$

$B^2 = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix}$

$B^3 = \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -9 & -11 \\ 22 & 13 \end{pmatrix}$

$B^4 = \begin{pmatrix} -9 & -11 \\ 22 & 13 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -31 & -24 \\ 48 & 17 \end{pmatrix}$

$B^5 = \begin{pmatrix} -31 & -24 \\ 48 & 17 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -79 & -41 \\ 82 & 3 \end{pmatrix}$

$B^6 = \begin{pmatrix} -79 & -41 \\ 82 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -161 & -44 \\ 88 & -73 \end{pmatrix}$

$B^{12} = \begin{pmatrix} -161 & -44 \\ 88 & -73 \end{pmatrix} \begin{pmatrix} -161 & -44 \\ 88 & -73 \end{pmatrix} = \begin{pmatrix} 22049 & 10296 \\ -20592 & 1457 \end{pmatrix}$

$B^n = \begin{pmatrix} a & b \\ -2b & c \end{pmatrix} \quad \forall a, b, c \in \mathbb{R}$

$$(3) A = \begin{pmatrix} -1 & 0 & 1 \\ 3 & k & 0 \\ -k & 1 & 4 \end{pmatrix}$$

$$a) \begin{pmatrix} -1 & 0 & 1 \\ 3 & k & 0 \\ -k & 1 & 4 \end{pmatrix} \xrightarrow[\substack{3F_1 + F_2 \\ kF_1 - F_3}]{} \begin{pmatrix} -1 & 0 & 1 \\ 0 & k & 3 \\ 0 & -1 & k-4 \end{pmatrix} \xrightarrow{F_2 + kF_3} \begin{pmatrix} -1 & 0 & 1 \\ 0 & k & 3 \\ 0 & 0 & 3+k^2-4k \end{pmatrix}$$

$$k^2 - 4k + 3 = 0 \begin{cases} k_1 = 3 \\ k_2 = 1 \end{cases}$$

Si $k=1,3$ no tiene inversa porque $\text{rg } A \neq \text{orden } A$

b) Si $k \neq 1,3 \rightarrow \text{rg } A = 3$

Si $k=1,3 \rightarrow \text{rg } A = 2$

$$(4) A = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c & d \\ -a+c & -b+d \end{pmatrix} = \begin{pmatrix} -b & a+b \\ -d & c+d \end{pmatrix}$$

$$\begin{cases} c = -b \\ d = a+b \\ -a+c = -d \\ -b+d = c+d \end{cases}$$

$$c = -b$$

$$\begin{pmatrix} a & b \\ -b & a+b \end{pmatrix} \quad \forall a, b \in \mathbb{R}$$

$$(5) \begin{cases} 2X + 3Y = A \\ 5X - 2Y = 2B \end{cases}$$

$$\begin{cases} \cdot (5) \quad 10X + 15Y = 5A \\ \cdot (-2) \quad -10X + 4Y = -4B \end{cases}$$

$$\begin{cases} \cdot (2) \quad 4X + 6Y = 2A \\ \cdot (3) \quad 15X - 6Y = 6B \end{cases}$$

$$Y = \frac{5A - 4B}{19}$$

$$X = \frac{2A + 6B}{19}$$

$$X = \frac{\begin{pmatrix} 2 & 0 & -2 \\ 4 & 6 & 8 \\ -2 & 2 & 0 \end{pmatrix} + \begin{pmatrix} 12 & 6 & -6 \\ 0 & 6 & 12 \\ 18 & -6 & 6 \end{pmatrix}}{19} = \frac{\begin{pmatrix} 14 & 6 & -8 \\ 4 & 12 & 20 \\ 16 & -4 & 6 \end{pmatrix}}{19}$$

$$Y = \frac{\begin{pmatrix} 5 & 0 & -5 \\ 10 & 15 & 20 \\ -5 & 5 & 0 \end{pmatrix} - \begin{pmatrix} 8 & 4 & -4 \\ 0 & 4 & 8 \\ 12 & -4 & 4 \end{pmatrix}}{19} = \frac{\begin{pmatrix} -3 & -4 & -1 \\ 10 & 11 & 12 \\ -17 & 9 & -4 \end{pmatrix}}{19}$$

$$X = \begin{pmatrix} 14/19 & 6/19 & -8/19 \\ 4/19 & 12/19 & 20/19 \\ 16/19 & -4/19 & 6/19 \end{pmatrix}$$

$$Y = \begin{pmatrix} -3/19 & -4/19 & -1/19 \\ 10/19 & 11/19 & 12/19 \\ -17/19 & 9/19 & -4/19 \end{pmatrix}$$