

CONTROL TEMA 1 2º BACH A

1. Sea la matriz $A = \begin{pmatrix} 0 & 1 & 1 \\ m & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

- Estudia el rango en función del valor de m .
- Calcular los valores de m para los cuales la matriz es singular.
- Si es posible, calcula la inversa de A para $m = -2$

2. Sea la matriz $B = \begin{pmatrix} -3 & 3 & 2 \\ -8 & 7 & 4 \\ 8 & -6 & -3 \end{pmatrix}$. Calcula B^n , B^{86} , B^{275}

3. Resuelve el sistema
$$\left. \begin{aligned} 2X + 5Y &= \begin{pmatrix} -5 & 3 \\ 16 & -6 \end{pmatrix} \\ 3X - 4Y &= \begin{pmatrix} 16 & -40 \\ 21 & 23 \end{pmatrix} \end{aligned} \right\}$$

4. Sea $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ -1 & -1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 1 & -1 \\ 3 & 0 & -1 \end{pmatrix}$. Resuelve $XA + X = C$

5. Calcula la matriz $A = \begin{pmatrix} a & b \\ c & 2 \end{pmatrix}$, con $b \neq 0$, que verifica que $A^2 = I$.

① $A = \begin{pmatrix} 0 & 1 & 1 \\ m & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

0,75 a) $\begin{pmatrix} 0 & 1 & 1 \\ m & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \xrightarrow{F_1 \leftrightarrow F_2} \begin{pmatrix} m & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \xrightarrow{F_1 - mF_3} \begin{pmatrix} m & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -m \end{pmatrix} \xrightarrow{F_2 - F_3}$

$\rightarrow \begin{pmatrix} m & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1+m \end{pmatrix}$ $1+m=0$
 Si $m \neq -1$ $\text{rg } A = 3$
 Si $m = -1$ $\text{rg } A = 2$

0,5 b) La matriz es singular, si no tiene inversa, luego será cuando $\text{rg } A \neq \text{orden}$
 Luego, no tendrá inversa si $m = -1$ y por lo tanto, será singular

0,75 c) Es posible calcular A^{-1} porque para $m = -2$ el $\text{rg } A = 3 = \text{orden } A$

$\left(\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{F_1 \leftrightarrow F_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array} \right) \xrightarrow{2F_1 + F_2}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array} \right) \xrightarrow{F_2 - F_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array} \right) \xrightarrow{\substack{F_1 - F_3 \\ F_2 - 2F_3}}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & 2 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array} \right) \Rightarrow A^{-1} = \begin{pmatrix} 1 & -1 & -1 \\ 2 & -1 & -2 \\ -1 & 1 & 2 \end{pmatrix}$

② $B = \begin{pmatrix} -3 & 3 & 2 \\ -8 & 7 & 4 \\ 8 & -6 & -3 \end{pmatrix}$

0,5 $B^2 = \begin{pmatrix} -3 & 3 & 2 \\ -8 & 7 & 4 \\ 8 & -6 & -3 \end{pmatrix} \begin{pmatrix} -3 & 3 & 2 \\ -8 & 7 & 4 \\ 8 & -6 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

0,75 $B^n = \begin{cases} n = 2k & I \\ n = 2k+1 & B \end{cases}$

0,75 $B^{86} = I$
 $B^{275} = B$

$$\textcircled{3} \quad \begin{cases} 2X+5Y = A \\ 3X-4Y = B \end{cases} \quad \left(\begin{array}{l} A = \begin{pmatrix} -5 & 3 \\ 16 & -6 \end{pmatrix} \\ B = \begin{pmatrix} 16 & -40 \\ 21 & 23 \end{pmatrix} \end{array} \right)$$

$$\bullet (3) \quad 6X + 15Y = 3A$$

$$\bullet (-2) \quad -6X + 8Y = -2B$$

$$\underline{23Y = 9A - 2B}$$

$$Y = \frac{9A - 2B}{23}$$

$$\bullet (4) \quad 8X + 20Y = 4A$$

$$\bullet (5) \quad 15X - 20Y = 5B$$

$$\underline{23X = 4A + 5B}$$

$$X = \frac{4A + 5B}{23}$$

$$X = \frac{4 \begin{pmatrix} -5 & 3 \\ 16 & -6 \end{pmatrix} + 5 \begin{pmatrix} 16 & -40 \\ 21 & 23 \end{pmatrix}}{23} = \begin{pmatrix} \frac{60}{23} & \frac{-182}{23} \\ \frac{169}{23} & \frac{91}{23} \end{pmatrix}$$

$$Y = \frac{3 \begin{pmatrix} -5 & 3 \\ 16 & -6 \end{pmatrix} - 2 \begin{pmatrix} 16 & -40 \\ 21 & 23 \end{pmatrix}}{23} = \begin{pmatrix} \frac{-47}{23} & \frac{89}{23} \\ \frac{6}{23} & \frac{-64}{23} \end{pmatrix}$$

$$\textcircled{4} \quad XA + X = C \rightarrow X(A+I) = C \rightarrow X = C \cdot (A+I)^{-1}$$

$$A+I = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ -1 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & 2 \\ -1 & -1 & 1 \end{pmatrix} = B$$

$$\left(\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{F_1+2F_3} \left(\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 0 & -3 & 2 & 1 & 0 & 2 \end{array} \right) \xrightarrow{\substack{2F_1+F_2 \\ 3F_2+2F_3}}$$

$$\xrightarrow{\substack{5F_1-F_3 \\ 5F_2-F_3}} \left(\begin{array}{ccc|ccc} 20 & 0 & 0 & 8 & 2 & -4 \\ 0 & 10 & 0 & -2 & 2 & -4 \\ 0 & 0 & 10 & 2 & 3 & 4 \end{array} \right) \xrightarrow{\substack{F_1/20 \\ F_2/10 \\ F_3/10}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/20 & 2/20 & -4/20 \\ 0 & 1 & 0 & -2/10 & 2/10 & -4/10 \\ 0 & 0 & 1 & 2/10 & 3/10 & 4/10 \end{array} \right)$$

$$X = \begin{pmatrix} 0 & 1 & -1 \\ 3 & 0 & -1 \end{pmatrix} \begin{pmatrix} 3/20 & 2/20 & -4/20 \\ -2/10 & 2/10 & -4/10 \\ 2/10 & 3/10 & 4/10 \end{pmatrix} = \begin{pmatrix} -4/10 & -1/10 & -8/10 \\ 1 & 0 & -1 \end{pmatrix}$$

$$\textcircled{5} \quad A^2 = I \quad \begin{pmatrix} a & b \\ c & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} a^2+bc & ab+2b \\ ac+2c & cb+4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} a^2+bc=1 \\ ab+2b=0 \\ ac+2c=0 \\ cb+4=1 \end{cases} \rightarrow \begin{cases} b(a+2)=0 \\ c(a+2)=0 \end{cases}$$

$b=0 \rightarrow a=-2$
 $c=0 \rightarrow$ No puede ser $cb+4 \neq 1$
 $a=-2$
 $bc = 1-4 = -3 \rightarrow c = -\frac{3}{b}$

$$\begin{pmatrix} -2 & b \\ -3/b & 2 \end{pmatrix} \quad \forall b \in \mathbb{R}$$