

**REC T.2 2º BACHILLERATO B**

1. Escribe las propiedades de los determinantes
2. Calcula el siguiente determinante:

$$\begin{vmatrix} -2 & 1 & -3 & 1 \\ 4 & -2 & 5 & -2 \\ 2 & -2 & 0 & 3 \\ 7 & -1 & 3 & -1 \end{vmatrix}$$

3. Sea la matriz

$$A = \begin{pmatrix} 2 & m-3 & 4 & 1 \\ m & -1 & 2 & 0 \\ -1 & m & -2m & m \end{pmatrix}$$

Estudia el rango de A en función de los valores de m.

4. Se consideran las matrices  $A = \begin{pmatrix} k-1 & 1 & -1 \\ 0 & k-2 & 1 \\ k & 0 & 2 \end{pmatrix}$ ;  $B = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -2 \\ 2 & 4 & 1 \end{pmatrix}$

- a. Obtener los valores de k para los que A es singular.
- b. Halle, si es posible, la matriz inversa de A en el caso de  $k=0$  y resuelva  $AX=B$

5. Sean A y B dos matrices cuadradas de orden 3, cuyos determinantes son  $|A| = 5$  y  $|B| = -3$ . Si  $A = (C_1, C_2, C_3)$ . Calcular:

a)  $|A^t B^4|$                       b)  $|A^{-1} \cdot 5B|$

c)  $|D| = |4C_2 + 3C_3 - C_1, -3C_2 + 5C_1, -7C_3 + 3C_1|$

$$\begin{array}{l}
 \textcircled{2} \quad \left| \begin{array}{cccc} -2 & \textcircled{1} & -3 & 1 \\ 4 & -2 & 5 & -2 \\ 2 & -2 & 0 & 3 \\ 7 & -1 & 3 & -1 \end{array} \right| = \left| \begin{array}{cccc} -2 & \textcircled{1} & -3 & 1 \\ 0 & 0 & -1 & 0 \\ -2 & 0 & -6 & 5 \\ 5 & 0 & 0 & 0 \end{array} \right| = - \left| \begin{array}{ccc} 0 & -1 & 0 \\ -2 & -6 & 5 \\ 5 & 0 & 0 \end{array} \right| = \\
 \begin{array}{l}
 F_2 = F_2 + 2F_1 \\
 F_3 = F_3 + 3F_1 \\
 F_4 = F_4 - F_1
 \end{array}
 \end{array}$$

$$= - [(-25) - 0] = 25$$

$$\textcircled{3} \quad A = \begin{pmatrix} 2 & m-3 & 4 & 1 \\ m & -1 & 2 & 0 \\ -1 & m & -2m & m \end{pmatrix}$$

$$\begin{aligned}
 |A| &= \begin{vmatrix} 2 & m-3 & 4 \\ m & -1 & 2 \\ -1 & m & -2m \end{vmatrix} = [4m + 4m^2 - 2(m-3)] - [4 + 4m - 2m^2(m-3)] = \\
 &= 4m + 4m^2 - 2m + 6 - 4 - 4m + 2m^3 - 6m^2 = 2m^3 - 2m^2 - 2m + 2 = 0
 \end{aligned}$$

$$\begin{array}{l}
 \left| \begin{array}{cccc} 2 & -2 & -2 & 2 \\ & 2 & 0 & -2 \\ 2 & 0 & -2 & 0 \end{array} \right| \quad 2m^2 - 2 = 0 \rightarrow m = \pm 1
 \end{array}$$

Si  $m \neq \pm 1$  el rango  $A = 3$

$$\text{Si } m=1 \quad \left( \begin{array}{cc|cc} 2 & -2 & 4 & 1 \\ 1 & -1 & 2 & 0 \\ -1 & 1 & -2 & 1 \end{array} \right) \quad \left| \begin{array}{ccc} -2 & 4 & 1 \\ -1 & 2 & 0 \\ 1 & -2 & 1 \end{array} \right| = -4 + 2 - 2 + 4 = 0 \Rightarrow \text{rg} = 2$$

$$\text{Si } m=-1 \quad \left( \begin{array}{cc|cc} 2 & -4 & 4 & 1 \\ -1 & -1 & 2 & 0 \\ -1 & -1 & 2 & -1 \end{array} \right) \quad \left| \begin{array}{ccc} 2 & -4 & 1 \\ -1 & -1 & 0 \\ -1 & -1 & -1 \end{array} \right| = (2+1) - (1-4) = 6 \Rightarrow \text{rg} = 3$$

Si  $m \neq \pm 1$   $\text{rg } A = 3$

Si  $m=1$   $\text{rg } A = 2$

Si  $m=-1$   $\text{rg } A = 3$

$$\begin{array}{l}
 \textcircled{4} \quad |A| = \begin{vmatrix} k-1 & 1 & -1 \\ 0 & k-2 & 1 \\ k & 0 & 2 \end{vmatrix} = 2(k-1)(k-2) + k + k(k-2) = \\
 \text{a) } \quad \quad \quad = 2k^2 - 6k + 4 + k + k^2 - 2k = 3k^2 - 7k + 4 = 0
 \end{array}$$

$$k = \frac{7 \pm \sqrt{49 - 48}}{6} = \begin{cases} 4/3 \\ 1 \end{cases}$$

Si  $k=1, 4/3 \rightarrow \text{rg } A \neq \text{orden} \Rightarrow \text{singular}$

$$b) \quad k=0 \quad A = \begin{pmatrix} -1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \quad |A| = 4$$

$$A_{11} = -4 \quad A_{21} = -2 \quad A_{31} = -1$$

$$A_{12} = 0 \quad A_{22} = -2 \quad A_{32} = 1$$

$$A_{13} = 0 \quad A_{23} = 0 \quad A_{33} = 2$$

$$A^{-1} = \frac{1}{4} \begin{pmatrix} -4 & -2 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$AX = B \rightarrow X = A^{-1}B$$

$$X = \frac{1}{4} \begin{pmatrix} -4 & -2 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -2 \\ 2 & 4 & 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -6 & -4 & -9 \\ 2 & 0 & 5 \\ 4 & 8 & 2 \end{pmatrix} = \begin{pmatrix} -3/2 & -1 & -9/4 \\ 1/2 & 0 & 5/4 \\ 1 & 2 & 1/2 \end{pmatrix}$$

$$⑤ \quad |A| = 5, \quad |B| = -3$$

$$a) \quad |A^t B^4| = |A^t| \cdot |B^4| = |A^t| \cdot |B|^4 = 5 \cdot (-3)^4 = 405$$

$$b) \quad |A^{-1} \cdot 5B| = |A^{-1}| \cdot |5B| = \frac{1}{|A|} \cdot 5^3 \cdot |B| = \frac{1}{5} \cdot 5^3 \cdot (-3) = -75$$

$$c) \quad |D| = |4C_2 + 3C_3 - C_1, -3C_2 + 5C_1, -7C_3 + 3C_1| =$$

$$= |4C_2, -3C_2 + 5C_1, -7C_3 + 3C_1| + |3C_3, -3C_2 + 5C_1, -7C_3 + 3C_1| +$$

$$+ |-C_1, -3C_2 + 5C_1, -7C_3 + 3C_1| =$$

$$= |4C_2, -3C_2, -7C_3 + 3C_1| + |4C_2, 5C_1, -7C_3 + 3C_1| + |3C_3, -3C_2, -7C_3 + 3C_1| +$$

$$+ |3C_3, 5C_1, -7C_3 + 3C_1| + |-C_1, -3C_2, -7C_3 + 3C_1| + |-C_1, 5C_1, -7C_3 + 3C_1| =$$

$$= |4C_2, 5C_1, -7C_3| + |4C_2, 5C_1, 3C_1| + |3C_3, -3C_2, 7C_3| + |3C_3, -3C_2, 3C_1| +$$

$$+ |3C_3, 5C_1, 7C_3| + |3C_3, 5C_1, 3C_1| + |-C_1, -3C_2, -7C_3| + |-C_1, -3C_2, 3C_1| =$$

$$= -140 |C_2, C_1, C_3| - 27 |C_3, C_2, C_1| - 21 |C_1, C_2, C_3| =$$

$$= 140 |A| + 27 |A| - 21 |A| = 146 |A| = 730$$