1. Dada la matriz $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 4 & 3 & k \end{pmatrix}$, Estudia el rango en función del valor de k. Calcula para qué valores de k existe la matriz inversa. Si es posible calcula la matriz inversa para k=6.

2. Calcula A^n, A^3, A^{21} *siendo* $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$

3. Dadas las matrices $A = \begin{pmatrix} -2 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ $y\ B = \begin{pmatrix} 4 & 1 & -1 \\ 0 & 2 & 1 \\ 2 & 0 & -3 \end{pmatrix}$.

   Resuelve el sistema matricial $\begin{cases} 3X - 4Y = A \\ 5X + 2Y = B \end{cases}$

4. Calcula todas las matrices B que conmutan con la matriz A=$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

5. Dadas las matrices A=$\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & -1 \\ 5 & -1 & 2 \end{pmatrix}$, B=$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, resuelve la siguiente ecuación $A \cdot X \cdot A^t = B$

TEMA 1

① Dada la matriz $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 4 & 3 & k \end{pmatrix}$. Estudia el rango en función del valor de k.
Calcula para qué valores de k existe la matriz inversa. Si es posible calcula la matriz inversa para $k=6$

- $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 4 & 3 & k \end{pmatrix} \xrightarrow[4F_1-F_3]{F_1-F_2} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 4-k \end{pmatrix} \xrightarrow{F_2 \leftrightarrow F_3} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 4-k \\ 0 & 0 & -1 \end{pmatrix}$

  El $rg\, A = 3 \quad \forall k \in \mathbb{R}$

- Existe la matriz inversa para cualquier valor de k, porque $rg\, A = 3 = \text{orden } A \quad \forall k \in \mathbb{R}$.

- Para $k=6$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 4 & 3 & 6 & 0 & 0 & 1 \end{array}\right) \xrightarrow[4F_1-F_3]{F_1-F_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 4 & 0 & -1 \end{array}\right) \xrightarrow{F_2 \leftrightarrow F_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 4 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{array}\right) \xrightarrow{F_1 - F_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & -3 & 0 & 1 \\ 0 & 1 & -2 & 4 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{array}\right) \xrightarrow[F_2-2F_3]{F_1+3F_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{array}\right) \xrightarrow{F_3/-1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 0 & -3 & 1 \\ 2 & 2 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

② Calcula A^n, A^3, A^{21}, siendo $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -I$$

$A^4 = A^3 \cdot A = -I \cdot A = -A$

$A^5 = A^4 \cdot A = -A \cdot A = -A^2$

$A^6 = A^3 \cdot A^3 = I$

$$A^n = \begin{cases} n = 6k \to I \\ n = 6k+1 \to A \\ n = 6k+2 \to A^2 \\ n = 6k+3 \to -I \\ n = 6k+4 \to -A \\ n = 6k+5 \to -A^2 \end{cases}$$

$A^{21} = A^3 = -I$

③ Dadas las matrices $A=\begin{pmatrix}-2&1&-1\\0&-1&1\\0&1&3\end{pmatrix}$ y $B=\begin{pmatrix}4&1&-1\\0&2&1\\2&0&-3\end{pmatrix}$.

Resuelve el sistema matricial $\left.\begin{matrix}3X-4Y=A\\5X+2Y=B\end{matrix}\right\}$

$$\left.\begin{matrix}3X-4Y=A\\5X+2Y=B\end{matrix}\right\} \quad \begin{matrix}\cdot(5)\\\cdot(-3)\end{matrix}\quad \begin{matrix}\cancel{15X}-20Y=5A\\\underline{-\cancel{15X}-6Y=-3B}\end{matrix}$$

$$-26Y=5A-3B \rightarrow Y=\frac{5A-3B}{-26}$$

$$\begin{matrix}\bullet & 3X-\cancel{4Y}=A\\ \bullet(2) & \underline{10X+\cancel{4Y}=2B}\\ & 13X=A+2B\end{matrix} \longrightarrow X=\frac{A+2B}{13}$$

$$X=\frac{1}{13}\left[\begin{pmatrix}-2&1&-1\\0&-1&1\\0&1&3\end{pmatrix}+\begin{pmatrix}8&2&-2\\0&4&2\\4&0&-6\end{pmatrix}\right]=\begin{pmatrix}6/13&3/13&-3/13\\0&3/13&3/13\\4/13&1/13&-3/13\end{pmatrix}$$

$$Y=-\frac{1}{26}\left[\begin{pmatrix}-10&5&-5\\0&-5&5\\0&5&15\end{pmatrix}-\begin{pmatrix}12&3&-3\\0&6&3\\6&0&-9\end{pmatrix}\right]=\begin{pmatrix}11/13&-1/13&1/13\\0&11/26&-1/13\\3/13&-5/26&-12/13\end{pmatrix}$$

④ Calcula todas las matrices B que conmutan con la matriz $A=\begin{pmatrix}1&2\\0&1\end{pmatrix}$

$$A\cdot B=B\cdot A \rightarrow \begin{pmatrix}1&2\\0&1\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1&2\\0&1\end{pmatrix}$$

$$\begin{pmatrix}a+2c&b+2d\\c&d\end{pmatrix}=\begin{pmatrix}a&2a+b\\c&2c+d\end{pmatrix}$$

$a+2c=a \rightarrow \boxed{c=0}$

$b+2d=2a+b \rightarrow 2a=2d \rightarrow \boxed{a=d}$

$c=c$

$d=2c+d \rightarrow \boxed{c=0}$

$$B=\begin{pmatrix}a&b\\0&a\end{pmatrix} \quad \forall a,b\in\mathbb{R}$$

⑤ Dadas las matrices $A=\begin{pmatrix}1&0&0\\-3&1&-1\\5&-1&2\end{pmatrix}$, $B\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$ resuelve $AXA^t=B$

$$AXA^t=B \longrightarrow X=A^{-1}\cdot B\,(A^t)^{-1}=A^{-1}\cdot B\cdot(A^{-1})^t$$

$$\left(\begin{array}{ccc|ccc}1&0&0&1&0&0\\-3&1&-1&0&1&0\\5&-1&2&0&0&1\end{array}\right)\xrightarrow[\substack{3F_1+F_2\\5F_1-F_3}]{}\left(\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&-1&3&1&0\\0&1&-2&5&0&-1\end{array}\right)\xrightarrow[F_2-F_3]{}\left(\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&-1&3&1&0\\0&0&1&-2&1&1\end{array}\right)$$

$$\xrightarrow[F_2+F_3]{}\left(\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&0&1&2&1\\0&0&1&-2&1&1\end{array}\right) \Longrightarrow A^{-1}=\begin{pmatrix}1&0&0\\1&2&1\\-2&1&1\end{pmatrix}$$

$$X=\begin{pmatrix}1&0&0\\1&2&1\\-2&1&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\begin{pmatrix}1&1&-2\\0&2&1\\0&1&1\end{pmatrix}=\begin{pmatrix}1&0&0\\1&-2&0\\-2&-1&0\end{pmatrix}\begin{pmatrix}1&1&-2\\0&2&1\\0&1&1\end{pmatrix}=$$

$$=\begin{pmatrix}1&1&-2\\1&-3&-4\\-2&-4&3\end{pmatrix}$$