TEMA 5. 1º BACHILLERATO A

1. a) Pasar a forma polar y trigonométrica: $-7+\sqrt{11}\,i$

   b) Pasar a forma binómica y trigonométrica: $\sqrt{7}_{128^{\circ}}$

   c) Pasar a forma binómica y polar z= 12(cos 50º+i sen 50º) (1,5 puntos)

2. Calcular m y n para que se cumpla:

   $\frac{-7n+3i}{8+mi}=\sqrt{7}_{120^{\circ}}$ (1,5 puntos)

3. Resuelve la ecuación $z^{12}=-3-8i$ (1,5 puntos)

4. Calcula:

   $a)\ \frac{(15+3i)(-2-5i)^8}{-6-3i}=$ b) $\frac{(4_{165^{\circ}}\cdot 7_{225^{\circ}})^6}{(11_{46^{\circ}})^4}=$ (1,5 puntos)

5. Resuelve (-2+3i)+(7-3i)8z= (z+1)(-5+i) - 2i

   (Hay que despejar z y tiene que ser un número complejo) (1 punto)

6. Calcula:

   $a)\ \frac{i^{27}-i^{18}+3i^{32}+i^{25}}{-4-i^{23}}=$ b) $\frac{-3i^{14}+7i^{15}+(-2i)^3}{4i^{15}-i^{24}}=$ (1,5 puntos)

7. La diferencia de dos números complejos conjugados es 16i y la suma de sus módulos es 20. Determina cuales son los dos números. (1,5 puntos)

TEMA 5 1°A

1) (1,1)

a) $z = -7 + \sqrt{11}\, i$

$r = \sqrt{49+11} = \sqrt{60}$

$\alpha = \operatorname{arctg} \frac{\sqrt{11}}{-7} = 154,65°$

Forma polar $z = \sqrt{60}_{154,65°}$

Forma trigonométrica $z = \sqrt{60}\,(\cos 154,65 + i \operatorname{sen} 154,65°)$

b) $z = \sqrt{7}_{128°}$

$a = \sqrt{7}\cos 128° = -1,63$

$b = \sqrt{7}\operatorname{sen} 128° = 2,08$

Forma binómica $z = -1,63 + 2,08\, i$

Forma trigonométrica $z = \sqrt{7}\,(\cos 128° + i \operatorname{sen} 128°)$

c) $z = 12\,(\cos 50° + i \operatorname{sen} 50°)$

$a = 12\cos 50° = 7,71$

$b = 12\operatorname{sen} 50° = 9,19$

Forma polar $z = 12_{50°}$

Forma binómica $z = 7,71 + 9,19\, i$

2) (1,5)

$\frac{-7n+3i}{8+mi} = \sqrt{7}_{120°} = -1,32 + 2,29i$

$(-7n+3i) = (8+mi)(-1,32 + 2,29i) = -10,56 + 18,32i - 1,32mi + 2,29mi^2$

$\left.\begin{array}{l} -7n = -10,56 - 2,29\,m \\ 3 = 18,32 - 1,32\,m \end{array}\right\}$

$m = \frac{3-18,32}{-1,32} = 11,60$

$n = \frac{10,56 - 2,29 \cdot 11,60}{-7} = 5,30$

$\boxed{m = 11,60}$

$\boxed{n = 5,30}$

3) (1,5)

$z^{12} = -3-8i$ $\qquad z = \sqrt{73}_{249,44°}$

$r = \sqrt{(-3)^2+(-8)^2} = \sqrt{9+64} = \sqrt{73}$

$\alpha = \operatorname{arctg} \frac{-8}{-3} = 249,44$

$s = \sqrt[12]{\sqrt{73}} = \sqrt[24]{73}$

$\beta = \begin{cases} 360° : 12 = 30° \\ 249,44° : 12 = 20,79° \end{cases}$

$\sqrt[24]{73}_{20,79°}$, $\sqrt[24]{73}_{50,79°}$, $\sqrt[24]{73}_{80,79°}$, $\sqrt[24]{73}_{110,79°}$, $\sqrt[24]{73}_{140,79°}$

$\sqrt[24]{73}_{170,79°}$, $\sqrt[24]{73}_{200,79°}$, $\sqrt[24]{73}_{230,79°}$, $\sqrt[24]{73}_{260,79°}$, $\sqrt[24]{73}_{290,79°}$

$\sqrt[24]{73}_{320,79°}$, $\sqrt[24]{73}_{350,79°}$

4)

a) $\frac{(15+3i)(-2-5i)^8}{(-6-3i)} = \frac{3\sqrt{26}_{11,31°}\,(\sqrt{29}_{248,20})^8}{\sqrt{45}_{206,57°}} = \frac{10819318,86_{1996,91}}{\sqrt{45}_{206,57°}} =$

$z = 15+3i = 3\sqrt{26}_{11,31°}$

$r = \sqrt{15^2+3^2} = \sqrt{634} = 3\sqrt{26}$

$\alpha = \operatorname{arc\,tg} \frac{3}{15} = 11,31°$

$z = -2-5i = \sqrt{29}_{248,20}$

$r = \sqrt{4+25} = \sqrt{29}$

$\alpha = \operatorname{arctg} \frac{-5}{-2} = 248,20°$

$z = -6-3i$

$r = \sqrt{36+9} = \sqrt{45}$

$\alpha = \operatorname{arc\,tg}\left(\frac{-3}{-6}\right) = 206,5$

$= 1612848,829_{1790,34} = 1612848,829_{350,34°}$

Escaneado con CamScanner

b) $\dfrac{(4_{165} \cdot 7_{225})^6}{(11_{46})^4} = \dfrac{(28_{390})^6}{14641_{184}} = \dfrac{481890304_{2340}}{14641_{184}} = 32913{,}75616_{356}$

5 (1)

$(-2+3i) + (7-3i)8z = (z+1)(-5+i) - 2i$

$-2+3i+56z-24iz = -5z+zi-5+i-2i$

$56z - 24iz + 5z - zi = -5 - i + 2 - 3i$

$61z - 25zi = -3-4i$

$z(61-25i) = -3-4i \rightarrow z = \dfrac{-3-4i}{61-25i} = \dfrac{(-3-4i)(61+25i)}{(61-25i)(61+25i)} =$

$= \dfrac{-183-75i-244i-100i^2}{3721-625i^2} = \dfrac{-83-319i}{4346} = -\dfrac{83}{4346} - \dfrac{319}{4346}i$

6 (1,5)

a) $\dfrac{i^{27} - i^{18} + 3i^{32} + i^{25}}{-4-i^{23}} = \dfrac{i^3 - i^2 + 3i^0 + i}{-4-i^3} = \dfrac{-i+1+3+i}{-4+i} = \dfrac{4}{-4+i} = \dfrac{4(-4-i)}{(-4+i)(-4-i)} =$

$= \dfrac{4(-4-i)}{16-i^2} = \dfrac{-16-4i}{17} = -\dfrac{16}{17} - \dfrac{4}{17}i$

b) $\dfrac{-3i^{14} + 7i^{15} + (-2i)^3}{4i^{15} - i^{24}} = \dfrac{-3i^2 + 7i^3 + (-8)i^3}{4i^3 - i^0} = \dfrac{3-7i+8i}{-4i-1} = \dfrac{3+i}{-1-4i} =$

$= \dfrac{(3+i)(-1+4i)}{(-1-4i)(-1+4i)} = \dfrac{-3+12i-i+4i^2}{1-16i^2} = \dfrac{-7+11i}{17} = -\dfrac{7}{17} + \dfrac{11}{17}i$

7 (1,5)

$z_1 = a+bi$ $\qquad z_1 - z_2 = 16i$

$z_2 = a-bi$ $\qquad |z_1| + |z_2| = 20$

$z_1 - z_2 = (a+bi) - (a-bi) = 2bi = 16i \rightarrow \boxed{b=8}$

$\sqrt{a^2+b^2} + \sqrt{a^2+(-b)^2} = 20 \Rightarrow \sqrt{a^2+64} + \sqrt{a^2+64} = 20 \rightarrow 2\sqrt{a^2+64} = 20$

$\sqrt{a^2+64} = 10 \rightarrow a^2+64 = 100 \rightarrow a = \pm\sqrt{36} \Rightarrow a = \pm 6$

Números son $z_1 = 6+8i$ $\qquad z_3 = -6+8i$

$z_2 = 6-8i$ $\qquad z_4 = -6-8i$