2° A

② $\begin{vmatrix} -2 & -1 & -3 & -1 & 4 \\ 2 & 3 & 5 & -2 & 0 \\ -1 & 1 & 0 & 1 & -2 \\ 5 & -1 & -4 & -2 & -1 \\ 3 & 0 & \textcircled{1} & -3 & 0 \end{vmatrix} \xrightarrow[C_4 = C_4 + 3C_3]{C_1 = C_1 - 3C_3} \begin{vmatrix} 7 & -1 & -3 & -10 & 4 \\ -13 & 3 & 5 & 13 & 0 \\ -1 & 1 & 0 & 1 & -2 \\ 17 & -1 & -4 & -14 & -1 \\ 0 & 0 & 1 & 0 & 0 \end{vmatrix} \longrightarrow \begin{vmatrix} 7 & -1 & -10 & 4 \\ -13 & 3 & 13 & 0 \\ -1 & \textcircled{1} & 1 & -2 \\ 17 & -1 & -14 & -1 \end{vmatrix}$

$\xrightarrow[C_3 = C_3 - C_2]{C_1 = C_1 + C_2} \begin{vmatrix} 6 & -1 & -9 & 2 \\ -10 & 3 & 10 & 6 \\ 0 & 1 & 0 & 0 \\ 16 & -1 & -13 & -3 \end{vmatrix} = (-1) \begin{vmatrix} 6 & -9 & 2 \\ -10 & 10 & 6 \\ 16 & -13 & -3 \end{vmatrix} = (-1)\left[(-180 + 260 - 864) - (320 - 468 - 270)\right] = -\left[-784 - (-418)\right] = -(-366) = 366$

③ $A = \left(\begin{array}{ccc|c} 1 & 3 & 4 & 1 \\ 1 & a & 2 & 2-a \\ -1 & 2 & a & a-2 \end{array}\right)$

$|A| = \begin{vmatrix} 1 & 3 & 4 \\ 1 & a & 2 \\ -1 & 2 & a \end{vmatrix} = (a^2 + 2) - (-a + 4) = a^2 + a - 2 = 0 \rightarrow a_1 = -2, \; a_2 = 1$

Si $a \neq 1, a \neq -2 \rightarrow$ rg $A = 3$

Si $a = 1$ $\left(\begin{array}{cc|cc} 1 & 3 & 4 & 1 \\ 1 & 1 & 2 & 1 \\ -1 & 2 & 1 & -1 \end{array}\right)$ $\quad \begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & 1 \\ -1 & 2 & -1 \end{vmatrix} = 0 \rightarrow$ rg $A = 2$

Si $a = -2$ $\left(\begin{array}{cc|cc} 1 & 3 & 4 & 1 \\ 1 & -2 & 2 & 4 \\ -1 & 2 & -2 & -4 \end{array}\right)$ $\quad \begin{vmatrix} 1 & 3 & 1 \\ 1 & -2 & 4 \\ -1 & 2 & -4 \end{vmatrix} = 0 \rightarrow$ rg $A = 2$

④ a) $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 1 & a-2 & a+1 \\ 1 & -1 & a^2+1 \end{vmatrix} = \left[(a-2)(a^2+1) - 2 - (a+1)\right] - \left[2(a-2) - (a+1) - (a^2+1)\right] =$

$= (a^3 - 2a^2 - 5) - (-a^2 + a - 6) = a^3 - a^2 - a + 1 = 0$

$(a-1)(a+1)(a-1) = 0$

0,5

Si $a \neq 1, a \neq -1 \Rightarrow |A| = 0 \Rightarrow \nexists$ inversa $\Rightarrow A$ es regular

b) $|A| = -2 - 3 + 3 + 5 = 3 \quad |A| = \begin{vmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ 1 & -1 & 5 \end{vmatrix}$

0,75

$A_{11} = 3 \quad A_{21} = 3 \quad A_{31} = -3$

$A_{12} = -2 \quad A_{22} = 3 \quad A_{32} = -1$

$A_{13} = -1 \quad A_{23} = 0 \quad A_{33} = 1$

$A^{-1} = \frac{1}{3} \begin{pmatrix} 3 & 3 & -3 \\ -2 & 3 & -1 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ -2/3 & 1 & -1/3 \\ -1/3 & 0 & 1/3 \end{pmatrix}$

e) $X - AX = B \rightarrow (I-A)X = B \longrightarrow X = (I-A)^{-1} \cdot B$

$$I - A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ 1 & -1 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 1 & -3 \\ -1 & 1 & -4 \end{pmatrix}$$

$|C| = |I-A| = (2+3) - (2+4) = -1$

$C_{11} = -1 \quad C_{21} = 2 \quad C_{31} = -1$

$C_{12} = -1 \quad C_{22} = -2 \quad C_{32} = 2$

$C_{13} = 0 \quad C_{23} = -1 \quad C_{33} = 1$

$$C^{-1} = \frac{1}{-1}\begin{pmatrix} -1 & 2 & -1 \\ -1 & -2 & 2 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 2 & -2 \\ 0 & 1 & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 2 & -2 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ 0 & -3 & -2 \\ 2 & 4 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 11 & 6 \\ -3 & -13 & -1 \\ -2 & -7 & -1 \end{pmatrix}$$

(5) a) $|A^{-1}| \cdot |B^5| = \frac{1}{|A|} \cdot |B|^5 = \frac{1}{-7}\left(\frac{2}{5}\right)^5 = \frac{2^5}{-7 \cdot 5^5} = -\frac{32}{21875}$

b) $|A^t\ 5B| = |A^t| \cdot |5B| = |A| \cdot 5^4 |B| = (-7) \cdot 625 \cdot \frac{2}{5} = -1750$

c) $|7C_3, -5C_1 + 6C_2, 7C_4 - 3C_3, -2C_1 - 8C_3| + |-2C_1, -5C_1 + 6C_2, 7C_4 - 3C_3, -2C_1 - 8C_3| +$

$+ |3C_2, -5C_1 + 6C_2, 7C_4 - 3C_3, -2C_1 - 8C_3| =$

$= |7C_3, -5C_1, 7C_4 - 3C_3, -2C_1 - 8C_3| + |7C_3, 6C_2, 7C_4 - 3C_3, -2C_1 - 8C_3| +$

$+ |-2C_1, -5C_1, 7C_4 - 3C_3, -2C_1 - 8C_3| + |-2C_1, 6C_2, 7C_4 - 3C_3, -2C_1 - 8C_3| +$

$+ |3C_2, -5C_1, 7C_4 - 3C_3, -2C_1 - 8C_3| + |3C_2, 6C_2, 7C_4 - 3C_3, -2C_1 - 8C_3| =$

$= |7C_3, -5C_1, 7C_4, -2C_1 - 8C_3| + |7C_3, -5C_1, -3C_3, -2C_1 - 8C_3| +$

$+ |7C_3, 6C_2, 7C_4, -2C_1 - 8C_3| + |7C_3, 6C_2, -3C_3, -2C_1 - 8C_3| +$

$+ |-2C_1, 6C_2, 7C_4, -2C_1 - 8C_3| + |-2C_1, 6C_2, -3C_3, -2C_1 - 8C_3| +$

$+ |3C_2, -5C_1, 7C_4, -2C_1 - 8C_3| + |3C_2, -5C_1, -3C_3, -2C_1 - 8C_3| =$

$= |7C_3, -5C_1, 7C_4, -2C_1| + |7C_3, -5C_1, 7C_4, -8C_3| + |7C_3, 6C_2, 7C_4, -2C_1| +$

$+ |7C_3, 6C_2, 7C_4, -8C_3| + |-2C_1, 6C_2, 7C_4, -2C_1| + |-2C_1, 6C_2, 7C_4, -8C_3| +$

$+ |-2C_1, 6C_2, -3C_3, -2C_1| + |-2C_1, 6C_2, -3C_3, -8C_3| + |3C_2, -5C_1, 7C_4, -2C_1| +$

$+ |3C_2, -5C_1, 7C_4, -8C_3| + |3C_2, -5C_1, -3C_3, -2C_1| + |3C_2, -5C_1, -3C_3, -8C_3| =$

$= -588\,|C_3, C_2, C_4, C_1| + 672\,|C_1, C_2, C_4, C_3| + 840\,|C_2, C_1, C_4, C_3| =$

$= -588\,|C_1, C_2, C_3, C_4| - 672\,|C_1, C_2, C_3, C_4| + 840\,|C_1, C_2, C_3, C_4| =$

$= -420\,|A| = 2940$