

$$\begin{pmatrix} -3 & 2 & 1 & 0 & -1 \\ 0 & -1 & 2 & 2 & 3 \\ -2 & 4 & 5 & 1 & -4 \\ 0 & -1 & -2 & -6 & 1 \\ 5 & 0 & 1 & -1 & 0 \end{pmatrix} \xrightarrow{\substack{C_1 - 5C_5 \\ C_4 + C_3}} \begin{pmatrix} -8 & 2 & 1 & 1 & -1 \\ -10 & -1 & 2 & 4 & 3 \\ -27 & 4 & 5 & 6 & -4 \\ 10 & -1 & -2 & -8 & 1 \\ 0 & 0 & 7 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -8 & 2 & 1 & -1 \\ -10 & -1 & 4 & 3 \\ -27 & 4 & 6 & -4 \\ 10 & -1 & -8 & 1 \end{pmatrix} \xrightarrow{\substack{F_1 + F_4 \\ F_2 - 3F_4 \\ F_3 + 4F_4}} \begin{pmatrix} 2 & 1 & -7 & 0 \\ -40 & 2 & 28 & 0 \\ 13 & 0 & -26 & 0 \\ 10 & -1 & -8 & 1 \end{pmatrix}$$

$$\xrightarrow{F_2 - 2F_1} \begin{vmatrix} 2 & 1 & -7 \\ -44 & 0 & 42 \\ 13 & 0 & -26 \end{vmatrix} = - \begin{vmatrix} -44 & 42 \\ 13 & -26 \end{vmatrix} = -(1144 - 546) = -598$$

$$(3) \quad A = \begin{pmatrix} a & 1 & 3 & 0 \\ 1 & a & 2 & 1 \\ 2 & 2a & 5 & a \end{pmatrix}$$

$$\begin{vmatrix} a & 1 & 3 \\ 1 & a & 2 \\ 2 & 2a & 5 \end{vmatrix} = 5a^2 + 6a + 4 - 6a - 4a^2 - 5 = a^2 - 1 = 0 \rightarrow a = \pm 1$$

Si $a \neq 1, -1$ $\text{rg } A = 3$

$$a = 1 \quad \begin{vmatrix} 1 & 1 & 3 & 0 \\ 1 & 1 & 2 & 1 \\ 2 & 2 & 5 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 3 & 0 \\ 1 & 2 & 1 \\ 2 & 5 & 1 \end{vmatrix} = 2 + 6 - 5 - 3 = 0 \quad \text{rg } A = 2$$

$$a = -1 \quad \begin{vmatrix} -1 & 1 & 3 & 0 \\ 1 & -1 & 2 & 1 \\ 2 & -2 & 5 & -1 \end{vmatrix} \quad \begin{vmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ -2 & 5 & -1 \end{vmatrix} = -2 - 6 - 5 - 3 \neq 0 \quad \text{rg } A = 3$$

$$(4) \quad A = \begin{pmatrix} -1 & 0 & 1 \\ 3 & k & 0 \\ -k & 1 & 4 \end{pmatrix} \quad |A| = -4k + 3 + k^2 = 0 \\ k^2 - 4k + 3 = 0 \quad k = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm 2}{2} = \begin{matrix} 3 \\ 1 \end{matrix}$$

a) Si $a = 1, 3$ es singular

$$b) \quad k = 0 \quad \begin{pmatrix} -1 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & 1 & 4 \end{pmatrix} \quad |A| = 3$$

$$\begin{matrix} A_{11} = 0 & A_{21} = 1 & A_{31} = 0 \\ A_{12} = -12 & A_{22} = -4 & A_{32} = 3 \\ A_{13} = 3 & A_{23} = 1 & A_{33} = 0 \end{matrix}$$

$$A^{-1} = \frac{1}{3} \begin{pmatrix} 0 & 1 & 0 \\ -12 & -4 & 3 \\ 3 & 1 & 0 \end{pmatrix}$$

$$X = A^{-1}B = \frac{1}{3} \begin{pmatrix} 0 & 1 & 0 \\ -12 & -4 & 3 \\ 3 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 0 & 3 \\ 2 & 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 3 \\ -30 & -24 \\ 9 & 6 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -10 & -8 \\ 3 & 2 \end{pmatrix}$$

$$(5) \quad a) \quad |A^t B^s| = |A^t| |B^s| = |A| \cdot |B|^s = \frac{2}{3} \cdot (-4)^5 = -\frac{2048}{3}$$

$$b) \quad |A^{-1} \cdot 6B| = \frac{1}{|A|} \cdot 6^3 |B| = \frac{3}{2} \cdot 6^3 (-4) = -\frac{2592}{2} = -1296$$

$$\begin{aligned}
 c) \quad |D| &= |3C_2, -2C_1 + 2C_2, 4C_3 - 6C_1| + |-5C_1, -2C_1 + 2C_2, 4C_3 - 6C_1| + |C_3, -2C_1 + 2C_2, 4C_3 - 6C_1| \\
 &= |3C_2, -2C_1, 4C_3 - 6C_1| + |3C_2, 2C_2, 4C_3 - 6C_1| + |-5C_1, -2C_1, 4C_3 - 6C_1| + |-5C_1, 2C_2, 4C_3 - 6C_1| \\
 &+ |C_3, -2C_1, 4C_3 - 6C_1| + |C_3, 2C_2, 4C_3 - 6C_1| = \\
 &= |3C_2, -2C_1, 4C_3| + |3C_2, -2C_1, -6C_1| + |-5C_1, 2C_2, 4C_3| + |-5C_1, 2C_2, -6C_1| + \\
 &+ |C_3, -2C_1, 4C_3| + |C_3, -2C_1, -6C_1| + |C_3, 2C_2, 4C_3| + |C_3, 2C_2, -6C_1| = \\
 &= -24 |C_2, C_1, C_3| - 40 |C_1, C_2, C_3| - 12 |C_3, C_2, C_1| = 24|A| - 40|A| + 12|A| = \\
 &= -4|A| = -4 \cdot \frac{2}{3} = -\frac{8}{3}
 \end{aligned}$$